Comment on "Quantum measurement and decoherence"

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Ford, Lewis, and O'Connell [Phys. Rev. A **64**, 032101 (2001)] have recently discussed a thought experiment in which a Brownian particle is subjected to a double-slit measurement . Analyzing the decay of the emerging interference pattern, they derive a decoherence rate that is much faster than previous results and even persists in the limit of vanishing dissipation. This result is based on the definition of a certain attenuation factor, which they analyze for short times. In this note, we point out that this attenuation factor captures the physics of decoherence only for times larger than a certain time t_{mix}, which is the time it takes until the two emerging wave packets begin to overlap. Therefore, the strategy of Ford *et al.* of extracting the decoherence time from the regime $t<t_{\mathrm{mix}}$ is in our opinion not meaningful. If one analyzes the attenuation factor for $t>t_{\mathrm{mix}}$, one recovers familiar behavior for the decoherence time; in particular, no decoherence is seen in the absence of dissipation. We confirm the latter conclusion by calculating the off-diagonal elements of the reduced density matrix.

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It is widely accepted that a rapid loss of coherence caused by the coupling to environmental degrees of freedom is at the root of the nonobservation of superpositions of macroscopically distinct quantum states. There is a now well-established theoretical scheme—"dissipative quantum mechanics"—for studying the details of this phenomenon, and for analyzing its time scale [2]. It has also become possible to observe decoherence in a variety of experiments in mesoscopic physics [3,4] and quantum optics [5–7].

In a recent publication [1], Ford, Lewis, and O'Connell (henceforth abbreviated as FLO) discuss a thought experiment in which a Brownian particle initially in thermal equilibrium with its environment is subjected to a double-slit position measurement, giving rise to an interference pattern. Analyzing the decay of this pattern, they derive a decoherence time that is much shorter than suggested by previous calculation [8]. They suggest the tentative explanation that inital particle-bath correlations, which drastically alter the short-time behavior of the Brownian particle, were not properly taken into account in previous work. Because the decoherence time calculated by FLO remains finite even in the absence of any coupling to the environment, they describe their result as "decoherence without dissipation" [9,10].

This is very puzzling. The usual physical picture of decoherence [2,11] is that averaging over unobserved degrees of freedom (the "environment") leads to nonunitary time evolution, with a consequent loss of information. If there is no coupling to the environment, there will be no such loss. This picture agrees with another commonly accepted definition of decoherence, namely, the decay of the off-diagonal elements of the reduced density matrix. Without environmental coupling, the time evolution of the system–and thus of ρ_{int}–is unitary, the norm of ρ_{int} is constant and does not decay.

In light of these obvious remarks, it is interesting to ask what FLO mean by decoherence. In this type of double-slit experiment it is essential that the two initially separated parts of the wave function eventually overlap if an interference pattern in the probability density of the particle is to be observed [12]. In the thought experiment considered by FLO, this overlap becomes sizeable only after the broadening of the two wave packets emerging from either slit becomes equal to their initial separation, which happens after a certain timescale t_{mix} to be defined in Eq. (3) below. On much shorter times, the interference pattern is influenced not only by the presence (or absence) of coherence, but also by the overlap of the wave functions, which makes it difficult, if not impossible, to extract from the probability density alone a measure of decoherence that is meaningful for $t<t_{\mathrm{mix}}$. As will be shown, the decoherence time obtained by FLO is much shorter than t_{mix} and hence merely reflects an arbitrariness in the definition of decoherence at these short times. Instead, the suppression of the interference pattern that FLO interpret as decoherence is due to the initial state being a mixed state at finite temperature.

Let us now give a brief summary of FLO's thought experiment [1] and thereby introduce some notation. A one-dimensional free Brownian particle, in thermal equilibrium with its environment, is suddenly (at time $t_1=0$, say) subjected to a double-slit position measurement, after which the state is described as

$$\rho_{\mathrm{ini}}(x,x',\{Q_\alpha\},\{Q'_\alpha\}) = \alpha^*(x)\alpha(x')\rho_{\mathrm{th}}(x,x',\{Q_\alpha\},\{Q'_\alpha\}), \tag{1}$$

where $\rho_{\mathrm{th}}(x,x',\{Q_\alpha\},\{Q'_\alpha\})$ denotes the density matrix of a particle (described by the coordinate x) in thermal equilibrium with its environment (described by a set of coordinates

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Q_α). The measurement function $\alpha(x)$ describes the transmittance of the double slit, is taken as a sum of two Gaussian functions with width $2\sigma_1$, separated by a distance $d \gg \sigma_1$, and is given by Eq. 15 of [1].

Above and hereafter, we chose units with $\hbar = k_B = 1$. After the mass, length, and energy scales are set by the particle mass m, the distance of the slits d, and by $E \equiv m^{-1}d^{-2}$, there are three remaining parameters: the slit width σ_1, the temperature T, and the friction coefficient of the Ohmic heat bath γ. In all quantities FLO consider, no divergencies are encountered as the bath cutoff is taken to infinity. In all plots below, we set the slit width as $\sigma_1 = d/20$.

FLO calculate the particle dynamics in the framework of a quantum Langevin equation [13,14], which describes a particle coupled to a dissipative environment with Ohmic characteristics. Within this framework, FLO calculate the probability density $P(x,t) = \tilde{\rho}(x,x,t)$ for finding the particle at time t at coordinate x, $\tilde{\rho}$ being the reduced density matrix of the Brownian particle. The result of this calculation, which we agree with and which we have reproduced using standard path integral techniques[15], is given by Eq. 16 of Ref. [1]. It can be written in the form

$$P(x,t) = P_{\rm cl}(x,t) + P_{\rm int}(x,t)\cos[\phi(x,t)], \tag{2}$$

where $P_{\rm cl}$ is the sum of the probabilities from each individual slit, and given by the second and third term in Eq. 16 of Ref. [1] (which we refer to as $P_{\rm cl}^-$ and $P_{\rm cl}^+$, respectively). Because the state is a superposition of the particle emanating from either slit, $P(x,t)$ also displays a spatial interference pattern $P_{\rm int}\cos\phi$ around $P_{\rm cl}$ (with a phase ϕ that will be of no further interest), given by the first term of Eq. 16 of Ref. [1]. We use in Eq. 16 of Ref. [1] $\sigma_2 = t_1 = 0$, and use the simplified expressions for $[x(0), x(t)]$ and $s(t)$ given after Eq. 19 of Ref. [1], valid for $\gamma \ll T$, which we assume from now on.

Before we comment on the further analysis of FLO, we shall briefly discuss some properties of the probability density $P(x,t)$. In Fig. 1(a), $P(x,t)$ is plotted for the parameters $\gamma = 0.3E$, $T = E$. The individual wave packets are seen to spread, and an interference pattern is seen to emerge only after the two wave packets, initially separated by the distance d, have developed a significant overlap, i.e., after their width has become equal to their separation d. For $T \ll Ed^2/\sigma_1^2$, which we assume from now on, the associated time scale $t_{\rm mix}$ is given by [15]

$$t_{\rm mix} \equiv 2m\sigma_1 d. \tag{3}$$

For $t < t_{\rm mix}$, the interference pattern is influenced not only by the loss of phase coherence, but mainly by the spreading of the wave packets, as is shown below. For $t > t_{\rm mix}$, the interference pattern is seen to broaden and to become flatter, as the wave function continues to spread.

In Fig. 1(b), the interference pattern for the parameters from Fig. 1(a) at time $t = t_{\rm mix}$ is compared to the case $T = \gamma = 0$. In Fig. 2, $P(x,t)$ is shown at three different times, together with the noninterfering part of the amplitude $P_{\rm cl}(x,t)$ and with the envelope of the interference pattern, given by

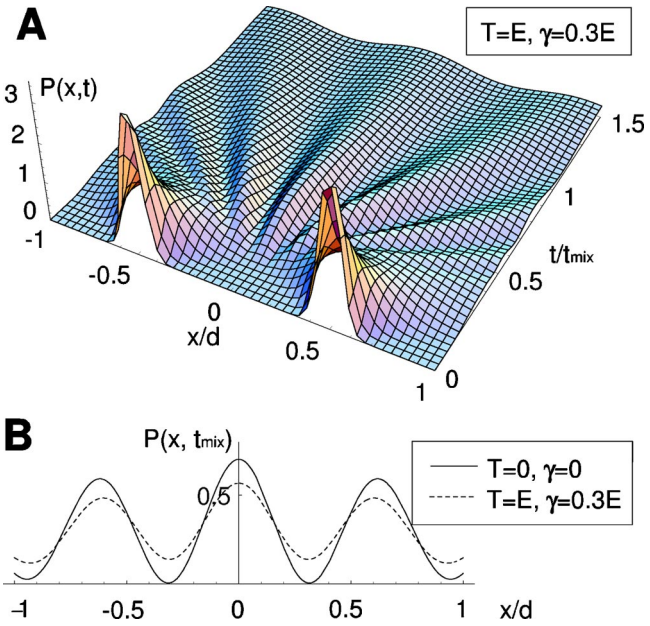

FIG. 1. (a) The probability density $P(x,t)$ for finding the particle at time t at coordinate x is plotted for $T = E$, $\gamma = 0.3E$. An interference fringe is seen to appear at time $t_{\rm mix}$. (b) $P(x, t = t_{\rm mix})$ is plotted for the same parameters as in (a) (dashed line), and for $T = \gamma = 0$ (solid line). The interference fringe in the former curve is seen to be somewhat suppressed with respect to the latter, but to be qualitatively very similar.

$P_{\rm cl}(x,t) \pm P_{\rm int}(x,t)$. As can be seen in Fig. 2, $P_{\rm cl}(x=0,t)$ is vanishingly small at $t \ll t_{\rm mix}$, and is rapidly growing as the wave packets start to overlap.

The interference fringes for both curves in Fig. 1(b) look quite similar. This is in drastic contrast to what one might expect from FLO's analysis, which leads to a decoherence time given by

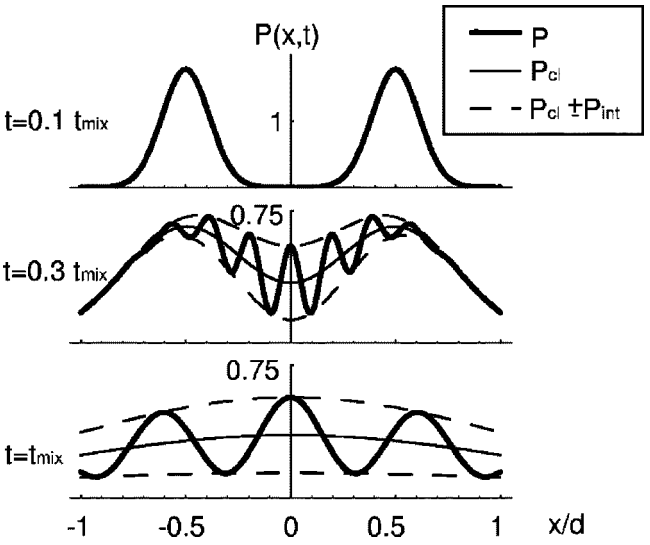

FIG. 2. $P(x,t)$ is shown at the times $t = 0.1 \times t_{\rm mix}$; $t = 0.3 \times t_{\rm mix}$; and $t = t_{\rm mix}$ (from top to bottom), for the parameters $T = E$, $\gamma = 0.3E$ (thick line). Also shown: The noninterfering contribution $P_{\rm cl}(x,t)$ (thin line) and the envelope $P_{\rm cl}(x,t) \pm P_{\rm int}(x,t)$ of the interference pattern (dashed line) around $P_{\rm cl}(x,t)$.

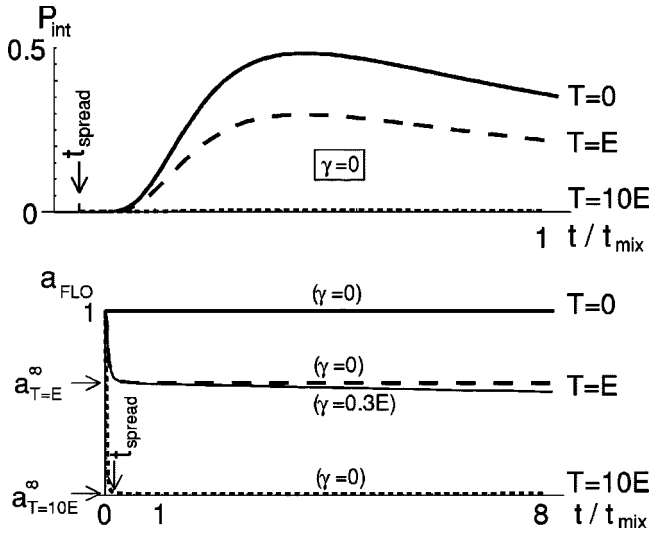

FIG. 3. Upper part: $P_{\rm int}(x=0,t)$ is shown for $\gamma=0$ and $T=0$ (solid; this line also coincides with that for $P_{\rm cl}(x=0,t)$ for all T shown), $T=E$ (dashed), and $T=10E$ (dotted). For small T, both $P_{\rm cl}$ and $P_{\rm int}$ grow rapidly after time $t_{\rm spread}$ (indicated by the arrow), after which broadening of the wave packets begins. The reduction of the interference pattern $P_{\rm int}$ with increasing T is due to the increasingly mixed-state nature of the initial state, not to decoherence, and has no time scale associated. In particular, at high temperatures ($T=10E$) $P_{\rm int}$ practically vanishes for *all* times. Lower part: the attenuation factor $a_{\rm FLO}=P_{\rm int}/P_{\rm cl}$ is plotted. Its decay from 1 down to a^{∞} takes place only for $t<t_{\rm spread}$. Also, for $T=E$ the case of no dissipation ($\gamma=0$) is compared to weak dissipation ($\gamma=0.3E$, thin solid line); a decoherence-induced decay of $a_{\rm FLO}$ for long times is seen in the latter case only.

$$\tau^{D}_{\rm FLO}=\frac{\sigma_1^2 m^{1/2}}{dT^{1/2}}. \tag{4}$$

The parameters for Fig. 1(a) imply, for example, $\tau^{D}_{\rm FLO}=0.025t_{\rm mix}$ [extracted from the very function $P(x,t)$]. We believe this value implies that by the time $t_{\rm mix}$, the entire interference pattern, clearly visible in Fig. 1, should have already disappeared. How can this be?

The decoherence analysis of FLO is based on an attenuation factor $a_{\rm FLO}(t)$, which is defined as [1,15]

$$a_{\rm FLO}(t)=\frac{P_{\rm int}(x,t)}{\sqrt{P^{+}_{\rm cl}(x,t)P^{-}_{\rm cl}(x,t)}}=\frac{P_{\rm int}(x=0,t)}{P_{\rm cl}(x=0,t)}. \tag{5}$$

In other words, $a_{\rm FLO}$ measures the interference amplitude $P_{\rm int}$ in units of the classical amplitude $P_{\rm cl}$ at $x=0$; hence it does not merely measure the time dependence of the interference pattern, but, via the denominator, also reflects the drastic increase of the reference unit $P_{\rm cl}(0,t)$ for $t<t_{\rm mix}$.

An example of $a_{\rm FLO}(t)$ is shown in Fig. 3, along with $P_{\rm cl}$ and $P_{\rm int}$. For low temperatures ($T\lesssim E$), $P_{\rm cl}$ and $P_{\rm int}$ are initially seen to be only slowly departing from their initial values $P_{\rm int}(t=0),P_{\rm cl}(t=0)\sim\exp(-d^2/8\sigma_1^2)$ ($\sim10^{-22}$ for the given parameters), and grow rapidly only after a time $t_{\rm spread}\equiv t_{\rm mix}\,\sigma_1/d$. As is shown in Ref. [15], this is because for $t<t_{\rm spread}$ the wave packet spreading is not effective yet. The rapid growth of $P_{\rm cl}(t)$ and $P_{\rm int}(t)$ by ~22 orders of magnitude seen in Fig. 3 takes place almost entirely between $t_{\rm spread}$ and $t_{\rm mix}$, when the overlap of the wave packets increases rapidly due to quantum spreading. On the other hand, the decrease of $a_{\rm FLO}$ takes place before $t_{\rm spread}$, when $P_{\rm int}$ and $P_{\rm cl}$ are still tiny. Indeed, $t<t_{\rm spread}$ is precisely the condition that $a_{\rm FLO}$ can be fitted by a Gaussian, $a_{\rm FLO}\approx\exp[-t^2/8(\tau^{D}_{\rm FLO})^2]$, from which $\tau^{D}_{\rm FLO}$ was extracted by FLO.

In the lower part of Fig. 3, the time evolution of $a_{\rm FLO}$ at finite temperature ($T=E$) is shown for vanishing dissipation $\gamma=0$, and compared to the case of finite $\gamma=0.3E$. For $\gamma=0$, $a_{\rm FLO}$ is seen to decay from the initial value $a_{\rm FLO}(t=0)=1$, and to saturate before $t_{\rm mix}$ at the value

$$a^{\infty}=\exp\left(-\frac{d^2}{8\sigma_1^2+2\lambda_{\rm th}^2}\right), \tag{6}$$

where $\lambda_{\rm th}^2=1/(mT)$ is the squared thermal wavelength.

During the preparation of our Comment and the subsequent Reply by Ford and O'Connell, they worked out some consequences of their theory for high temperatures ($T\gg E$, or $\lambda_{\rm th}\ll d$). We have likewise generalized our results, to make possible a meaningful comparison between the conclusions drawn by each group. As is pointed out after Eq. (20) of Ref. [1], a^{∞} is highly suppressed at high temperatures. FLO interpret this as decoherence. As we show below, however, the suppression of $a_{\rm FLO}$ is merely due to the initial state being a mixed one, and has no time scale associated with it, contrary to what the time evolution of $a_{\rm FLO}$ might suggest. For example, the interference amplitude $P_{\rm int}$ in Fig. 3 vanishes for *all* times if $T\gg E$.

A time scale is only introduced if $\gamma>0$: In this case, $a_{\rm FLO}$ is further reduced in a time-dependent way and assumes for $t\gg\max(1/\gamma,t_{\rm mix})$ the simple limiting form

$$a_{\rm FLO}(t)\to\exp\left(-\frac{t}{t_{\rm dec}(1+t/t_s)}\right), \tag{7}$$

where $t_{\rm dec}=\lambda_{\rm th}^2/(d^2\gamma)$, and $t_s=t_{\rm dec}d^2/(8\sigma_1^2)\gg t_{\rm dec}$. For $t_{\rm mix}<t\ll t_s$, $a_{\rm FLO}$ decays exponentially on the decoherence time scale $t_{\rm dec}$ (this was also found by FLO in the long time limit).

Because the overlap of the two wave packets is more or less constant for $t>t_{\rm mix}$, the amplitude of the interference pattern $P_{\rm int}(x,t)$, and thus $t_{\rm dec}$, is a meaningful measure of their phase coherence. Indeed, interference patterns have been analyzed in this way, most explicitly in Refs. [5] and [7]. The decoherence time $t_{\rm dec}$ agrees with what one expects on general grounds [16], and what was observed experimentally in a somewhat similar context [5]. Note that $t_{\rm dec}$ diverges as γ or T vanish, such that no decoherence without dissipation is seen.

For $t\gg t_s$, $a_{\rm FLO}$ is found to saturate at the (tiny) value $a_{\rm FLO}(t\to\infty)=\exp[-d^2/(8\sigma_1^2)]$. This is probably related to the small initial overlap of the wave packets, and is further discussed in Ref. [15].

So far, we have argued that $a_{\rm FLO}$ is not suitable to reveal meaningful information about decoherence at $t<t_{\rm mix}$. How, then, would one obtain such information? Since this question cannot be answered using the diagonal elements of the re-

duced density matrix $\tilde{\rho}$ alone, let us consider the entire reduced density matrix, including its off-diagonal elements. A definition of decoherence valid for all times including $t<t_{\mathrm{mix}}$ has been proposed in Ref. [17]. It relies on the observation that $\tilde{\rho}$ splits naturally into a classical part $\tilde{\rho}_{\mathrm{cl}}$ and an interference part $\tilde{\rho}_{\mathrm{int}}$ ($\tilde{\rho}=\tilde{\rho}_{\mathrm{cl}}+\tilde{\rho}_{\mathrm{int}}$), which produce the corresponding terms in the probability density in Eq. (2). As is seen explicitly in Eq. (A10) of Ref. [15], the former consists of peaks around the diagonal $\tilde{\rho}(\pm d/2,\pm d/2)$, the latter around the off-diagonal of $\tilde{\rho}(\pm d/2,\mp d/2)$ of the reduced density matrix. Therefore, the norm $a_{\mathrm{OD}}(t)$, defined in Ref. [17] by

$$|a_{\mathrm{OD}}(t)|^2=\mathrm{Tr}\ \tilde{\rho}_{\mathrm{int}}(t)\tilde{\rho}_{\mathrm{int}}(t)^{\dagger}, \tag{8}$$

describes the temporal fate of the interference term even for $t<t_{\mathrm{mix}}$, i.e., before it appears in the diagonal of $\tilde{\rho}$. Very importantly, the dissipationless case $\gamma=0$ describes a closed system with unitary time evolution, $\tilde{\rho}(t)=U\tilde{\rho}U^{\dagger}$ with $U^{-1}=U^{\dagger}$; hence Eq. (8) is in this case automatically independent of time. This is the back-of-the-envelope "proof" (already given, e.g., in Ref. [17]) that there can be no decoherence without dissipation.

Further insight is gained by analyzing a_{OD} for $\gamma=0$. A simple calculation [15], using results from Ref. [18]; gives

$$a_{\mathrm{OD}}(t)=\frac{1}{\sqrt{2}}\exp\left(-\frac{d^2}{2\lambda_{\mathrm{th}}^2}\right)=\frac{1}{\sqrt{2}}a^{\infty}. \tag{9}$$

(Only the temperature dependence of a_{OD} is important; the trivial factor $1/\sqrt{2}$ would disappear if a_{OD} was normalized differently.) As was seen on general grounds, this is indeed *independent* of time. Consequently, in the absence of dissipation, *no time scale* is associated with the reduction of the attenuation factor a^{∞} below 1 as the temperature is increased. Instead, this reduction is already present in the initial state, which is not a pure state for $T\neq 0$, but a mixed state with a momentum uncertainty of the order of $\lambda_{\mathrm{th}}^{-1}$. This reduces $|\tilde{\rho}_{\mathrm{int}}|^2$ and thereby the interference pattern precisely by the factor a^{∞} [15]. Incidentally, a^{∞} is exponentially small in d/λ_{th}, which FLO chose to be $\gg 1$. Therefore, for their choice of parameters, the interference pattern for $\gamma=0$ is exponentially small for *all* times. Of course, the fact that the initial state is a mixed state should not be confused with decoherence: The latter is a dynamical process with an associated time scale, the former is not.

In conclusion, we have shown in this work, using FLO's own formulas, that for short times $t<t_{\mathrm{mix}}$, the measure of decoherence suggested by FLO does not permit the change in overlap of the wave packets to be distinguished from the decay of the interference pattern, and therefore has nothing to say about decoherence. Moreover, a simple calculation of the attenuation factor based on the off-diagonal elements of the reduced density matrix at $\gamma=0$ clearly shows that it does not depend on time at all. These problems with FLO's interpretation of their result exist for all choices of parameters, in particular for all temperatures.

Note added. From a private communication with O'Connell, we learned that Murakami, Ford, and O'Connell [19] have themselves recently concluded that in the absence of dissipation, "there is no decoherence in (Wigner) phase space." We fully agree with this conclusion, but (contrary to Ref. [19]) believe it to be inconsistent with FLO's earlier claims of decoherence without dissipation (in coordinate space). The resolution of this inconsistency is that FLO's measure of decoherence in coordinate space is meaningless in the short-time limit, as argued above.

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