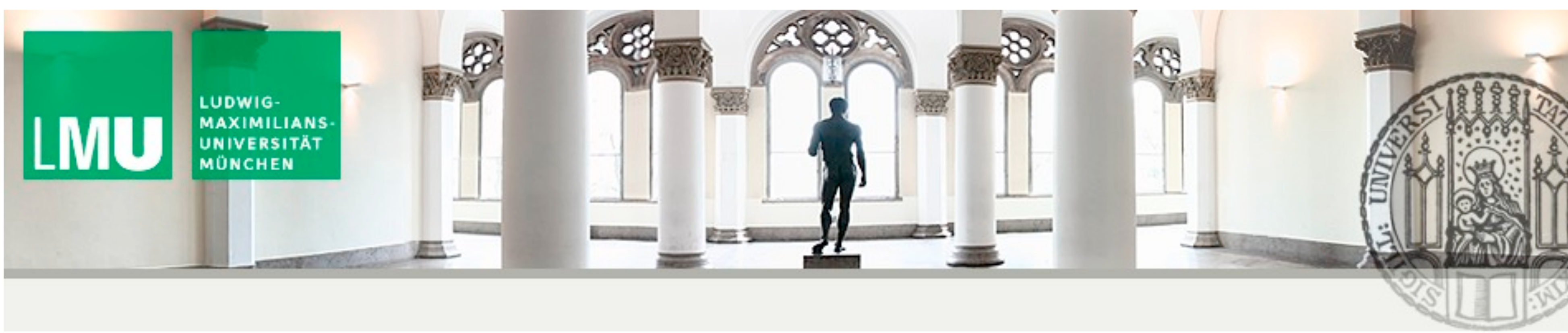
String Amplitudes for the LHC in D-brane Compactifications

Dieter Lüst, LMU (Arnold Sommerfeld Center)
and MPI München



Dublin 6. April 2009

Introduction:

Count the number of consistent string vacua ➤

Vast landscape with $N_{sol} = 10^{500-1500}$ vacua!

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Two strategies to find something interesting:

- Explore all mathematically consistent possibilities: top down approach (quite hard), string statistics.

- Do not look randomly - look for green (promising) spots in the landscape ➠ model building, bottom up approach.

Anthropic principle?

Our universe is not special!

(Susskind, 2003; see also Schellekens, arXiv:0807.3249)

Observed parameters take their observed values for the simple reason that they allow for intelligent life.

- Fine structure and strong coupling constants: nucleo-synthesis

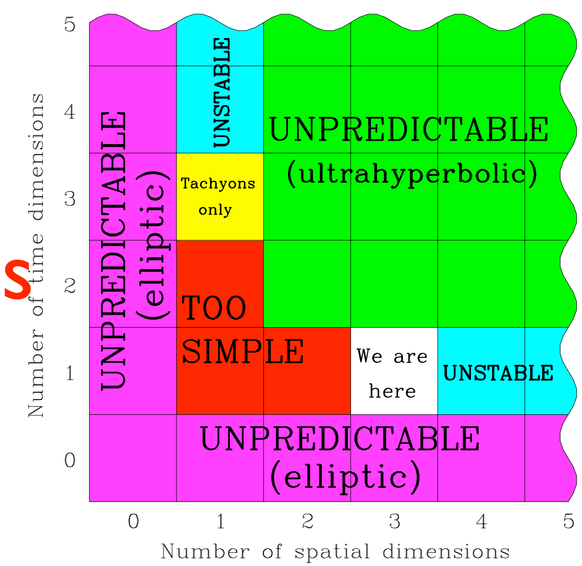
  Gravity:

- Fine tuning of cosmological constant: $\Lambda/M_{\rm Planck}^4 \simeq 10^{-120}$

  (Weinberg, 1987)

  $\Rightarrow$ Need at least 10^{120} vacua!

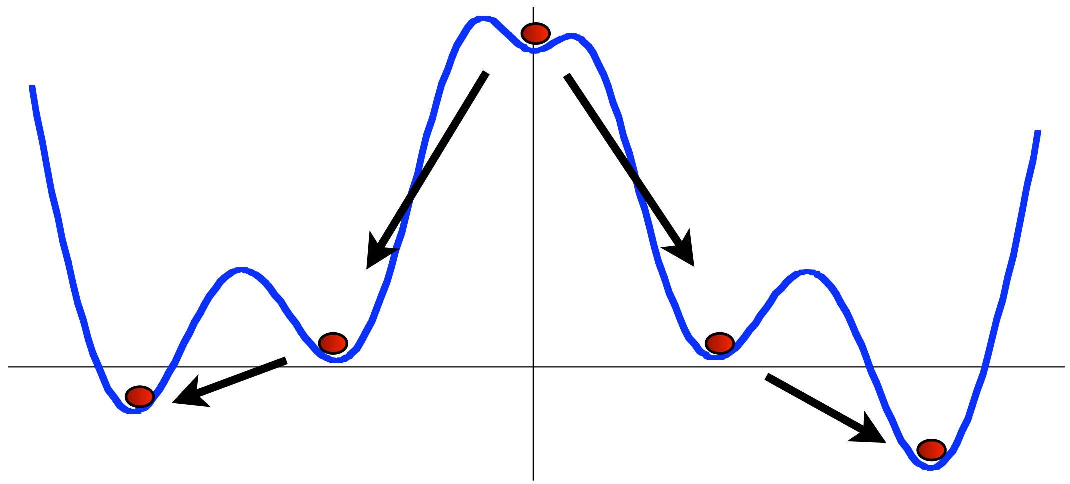
- Ehrenfest: number of spatial dimensions



Multiverse picture: (Linde, 1986)

Transition amplitudes between different vacua (wave function of the universe):

(Hartle, Hawking, 1983)



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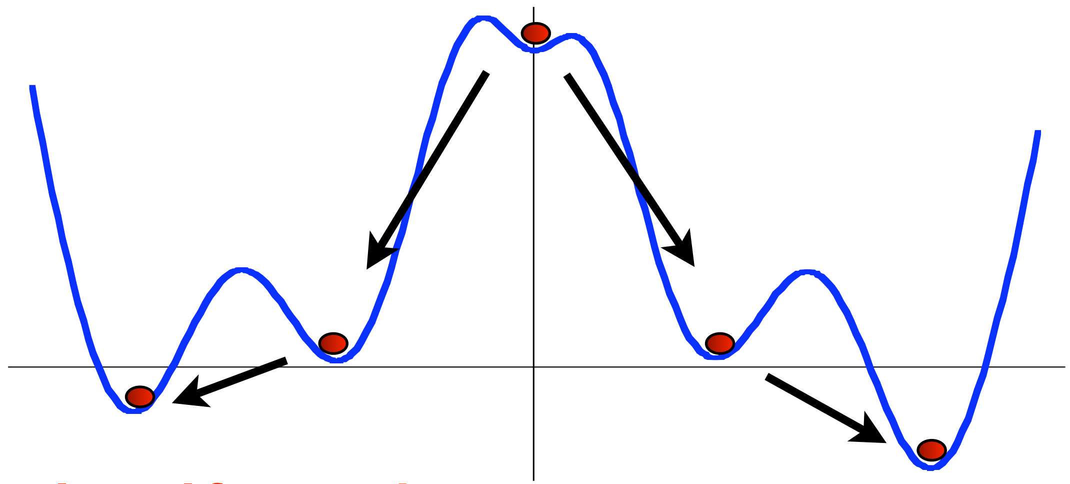
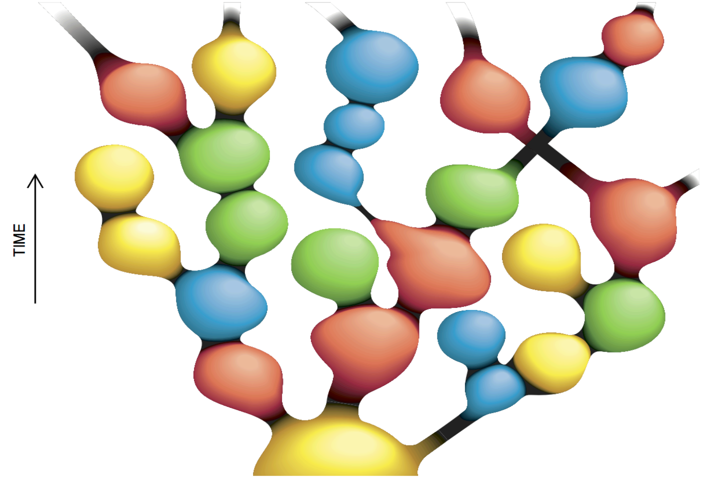


⇨ Eternal, self-producing universe:



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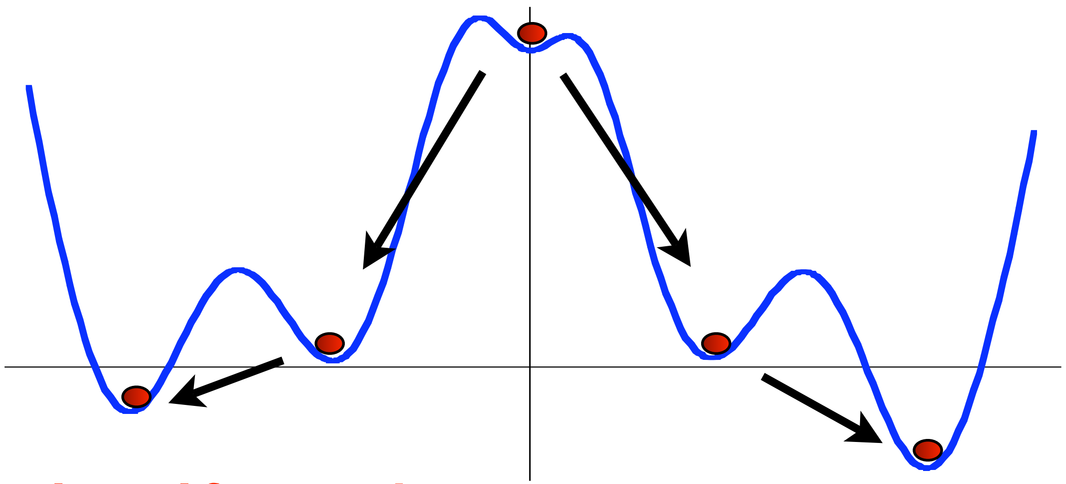

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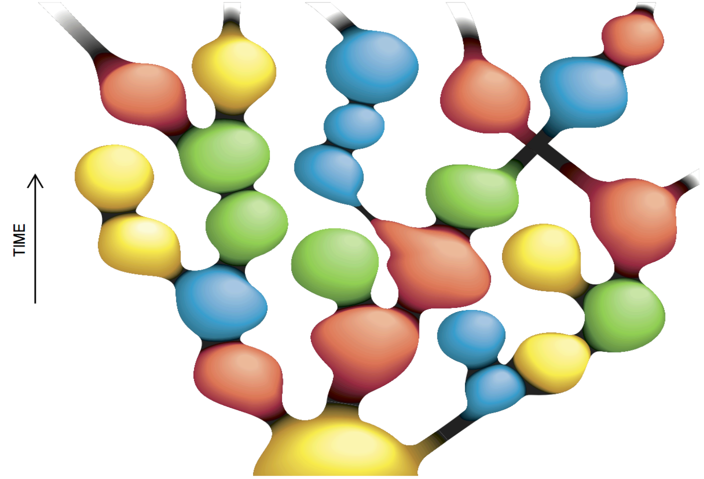

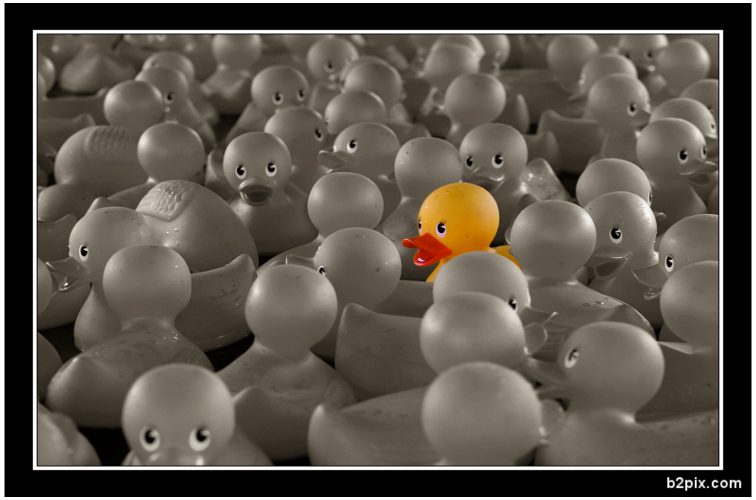

General feature of string theory:

Geometrization of particles and their interactions!

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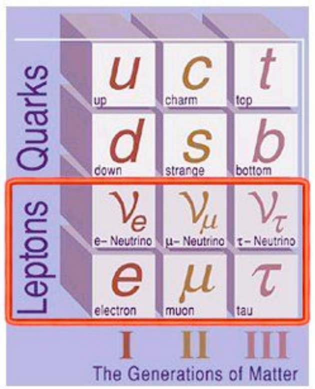
Dictionary:

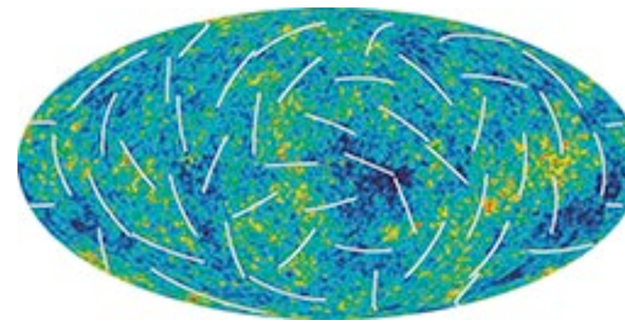
Particles physics



Gauge interactions:

$$G = SU(3) \times SU(2) \times U(1)$$

Cosmology



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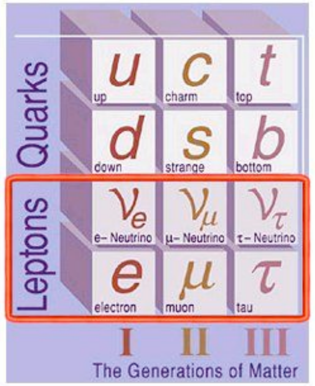

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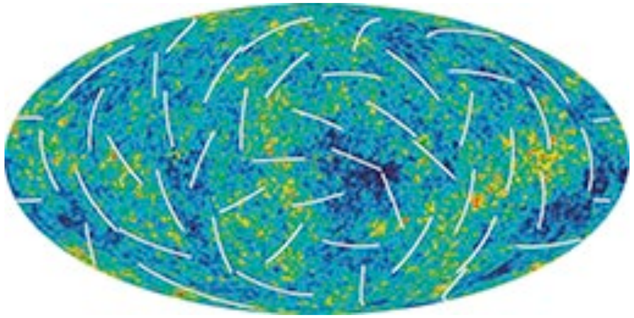
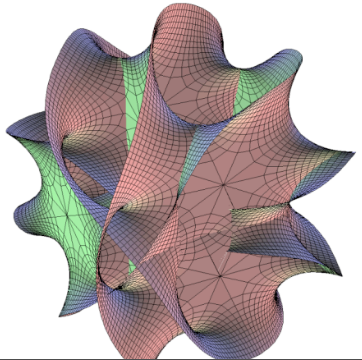


geometry & topology of strings and branes



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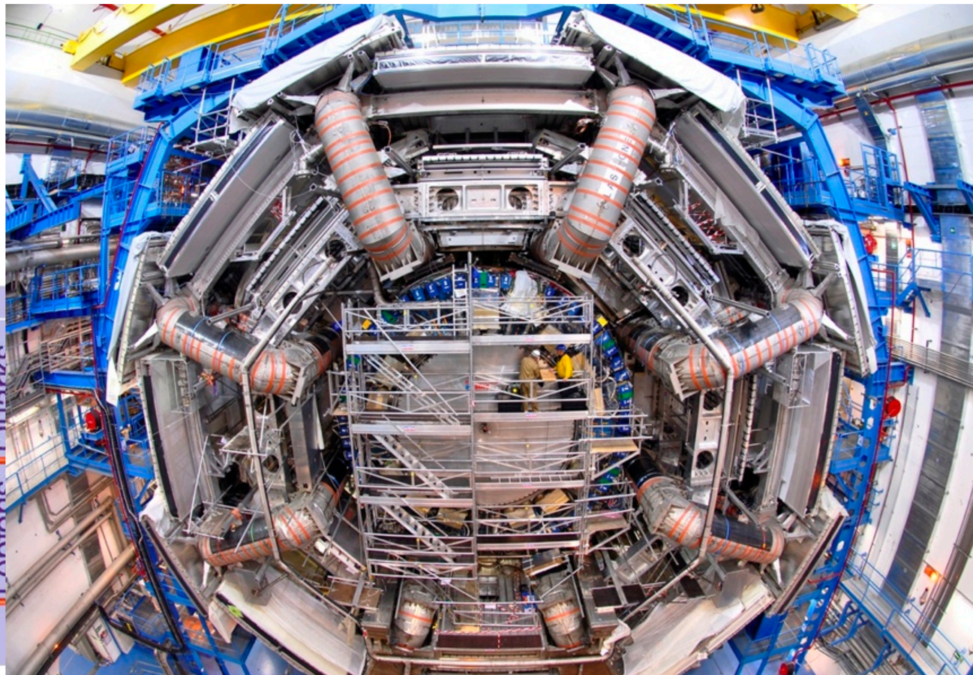

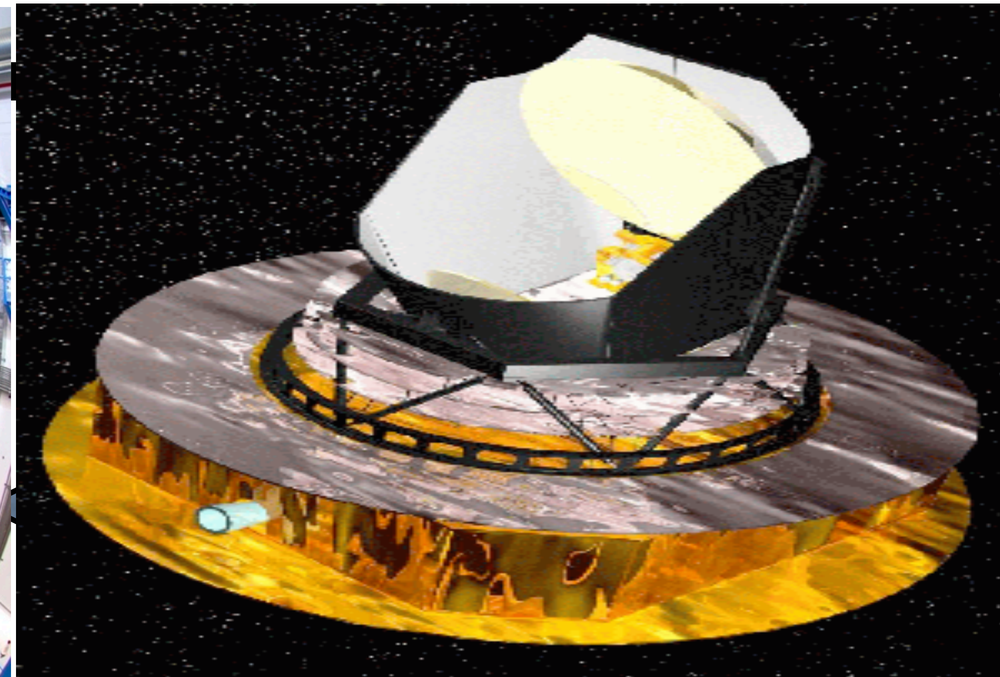

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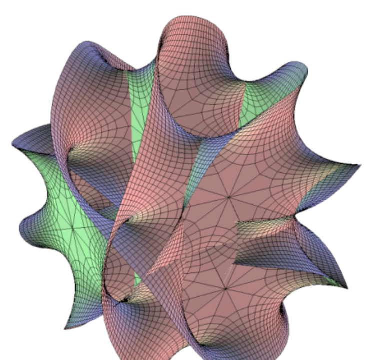

Strategy for string phenomenology:

Consider (only) those vacua that realize the Standard Model
(by-pass the landscape problem):

- What is the likelihood for vacua with the SM-like properties?
- What are their generic, model independent features?
- Can we make model independent predictions beyond the SM?
- Can we test these predictions in experiments (LHC)?

Bottum-up approach has to meet
top-down approach!

Outline

- Intersecting D-brane models
- Mass scales in D-brane models
- Stringy amplitudes for the LHC

(The LHC string hunter's companion)

II) (Intersecting) D-brane models:

(Bachas (1995); Blumenhagen, Görlich, Körs, Lüst (2000); Angelantonj, Antoniadis, Dudas Sagnotti (2000); Ibanez, Marchesano, Rabadan (2001); Cvetic, Shiu, Uranga (2001); ...)

Alternative constructions: heterotic strings

(Braun, He, Ovrut, Pantev; Bouchard, Donagi; Buchmüller, Hamaguchi, Lebedev, Nilles, Ramos-Sanchez, Ratz, Vaudrevange; Groot Nibbelink, Held, Ruehle, Trapletti, Vaudrevange; Faraggi, Kounnas, Rizos)

F-theory (Beasly, Heckman, Marsano, Saulina, Schafer-Nameki, Vafa; Donagi, Wijnholt, ...)

Consider open string compactifications with intersecting D-branes ➠ Type IIA/B orientifolds:

Features:

- Non-Abelian gauge bosons live as open strings on lower dimensional world volumes π of D-branes.
- Chiral fermions are open strings on the intersection locus of two D-branes: $N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b) \equiv \pi_a \circ \pi_b$

Perturbative type II orientifolds contain:

(Review: Blumenhagen, Körs, Lüst, Stieberger, hep-th/0610327)

- Closed string 6-dimensional background geometry:

  -Torus, orbifold, Calabi-Yau space,
  generalized spaces with torsion.

- Space-time filling D(3+p)-branes wrapped around internal p-cycles:

  - Open string matter fields.

- Strong consistency conditions:

  - tadpole cancellation with orientifold planes.

Intersecting D6-brane models:

D6 wrapped on 3-cycles π_a , intersect at angles θ_{ab}

Tadpole condition: $$\sum_a N_a \pi_a = \pi_{O6}$$

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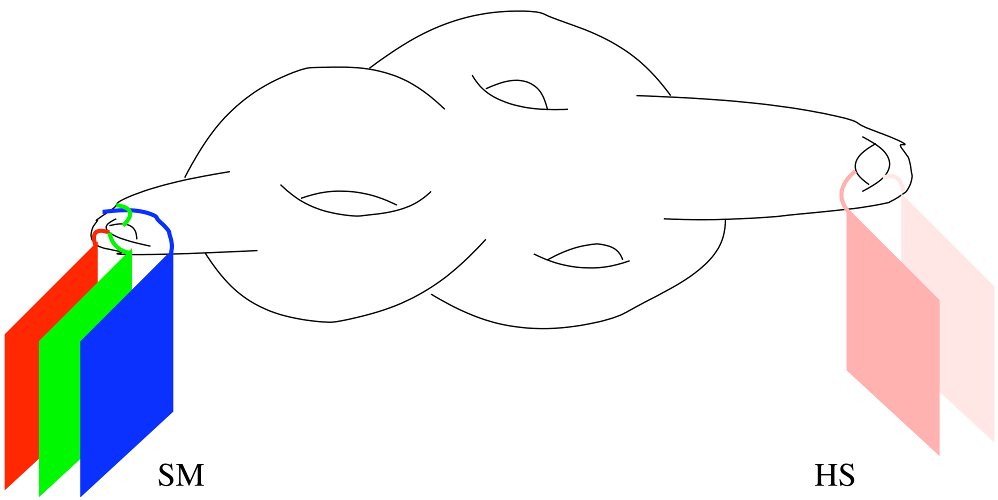

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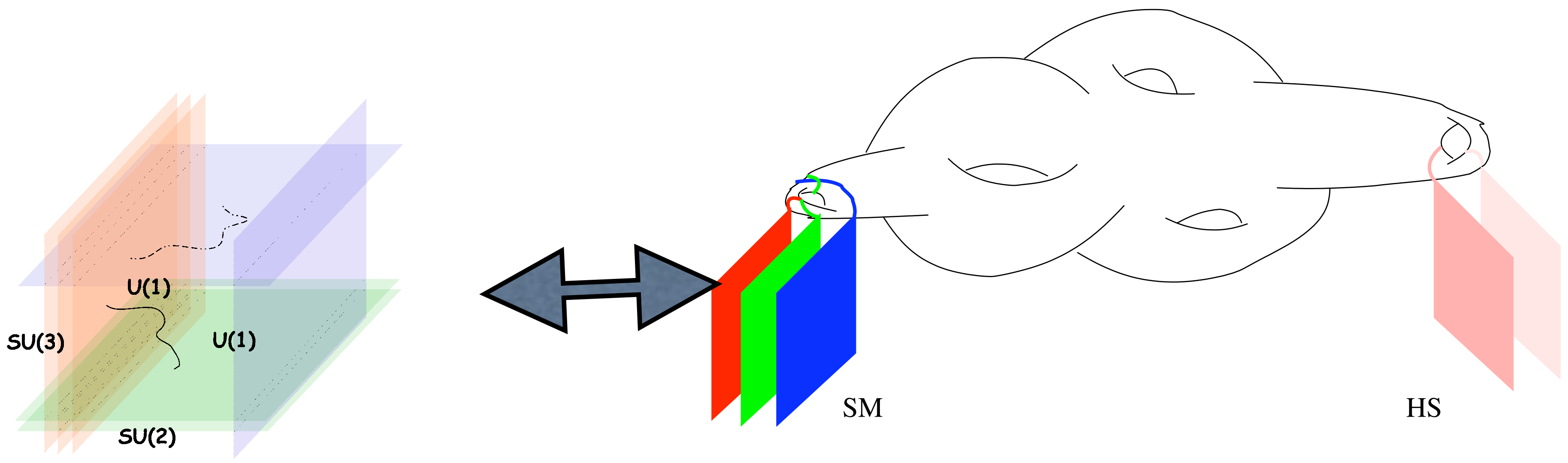

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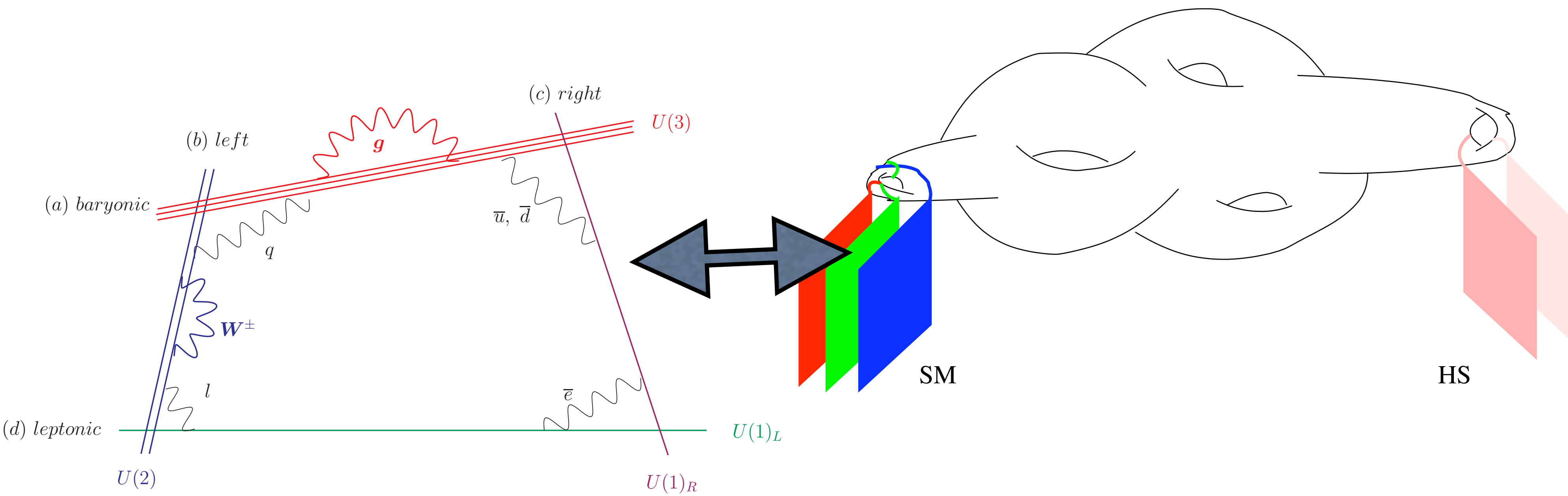

(Ibanez,Marchesano, Rabadan, hep-th/0105155;
Blumenhagen, Körs, Lüst, Ott, hep-th/0107138)

(Intersecting) D6-brane statistics

How many orientifold models exist which come close to the (spectrum of the) MSSM?

(Blumenhagen, Gmeiner, Honecker, Lüst, Stein, Weigand; related work: Dijkstra, Huiszoon, Schellekens, hep-th/0411129; Anastasopoulos, Dijkstra, Kiritsis, Schellekens, hep-th/0605226; Douglas, Taylor, hep-th 0606109; Dienes, Lennek, hep-th/0610319)

Example: $\mathcal{M}_6 = T^6/(Z_N \times Z_M)$ IIA orientifold:

Systematic computer search (NP complete problem):

Look for solutions of a set of diophantic equations:

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Z6'-orientifold: (Gmeiner, Honecker, arXiv:0806.3039)

Millions of standard models!

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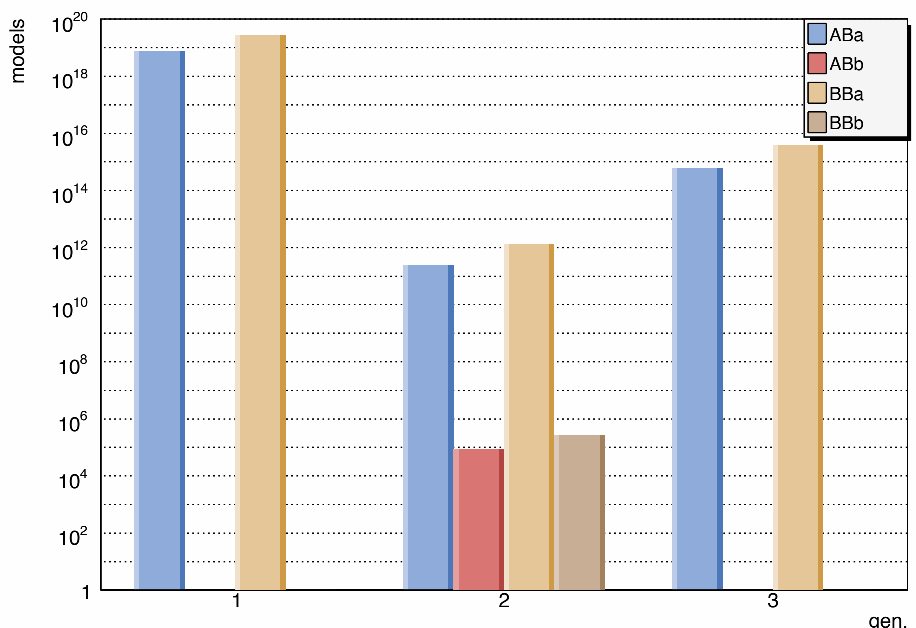

SM gauge group with # of generations

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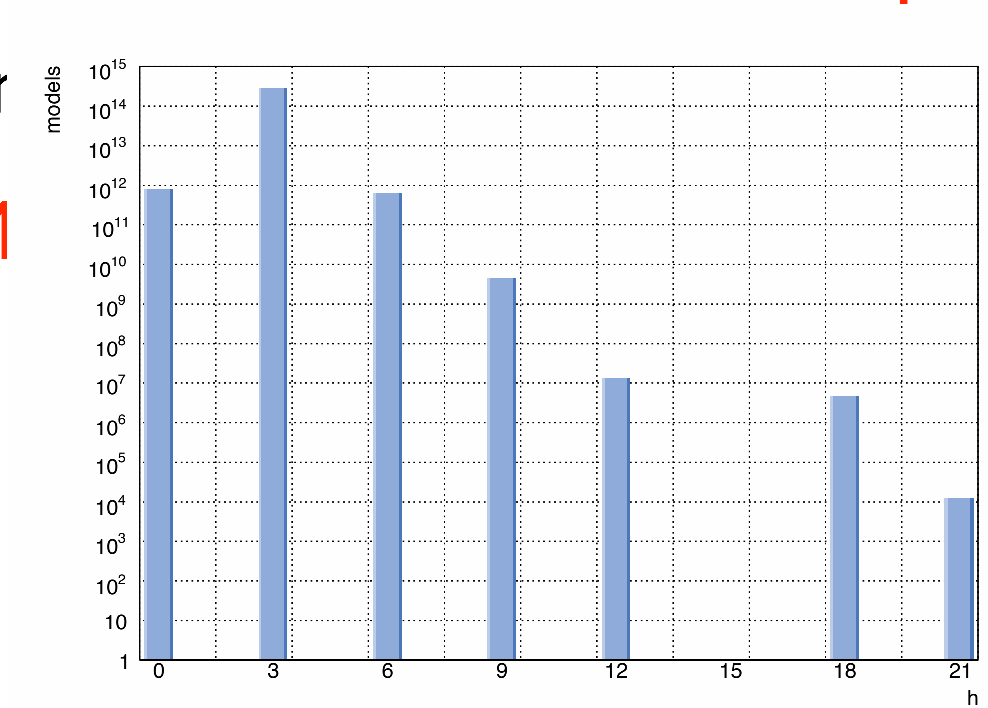

SM gauge group, 3 generations with # of Higgses

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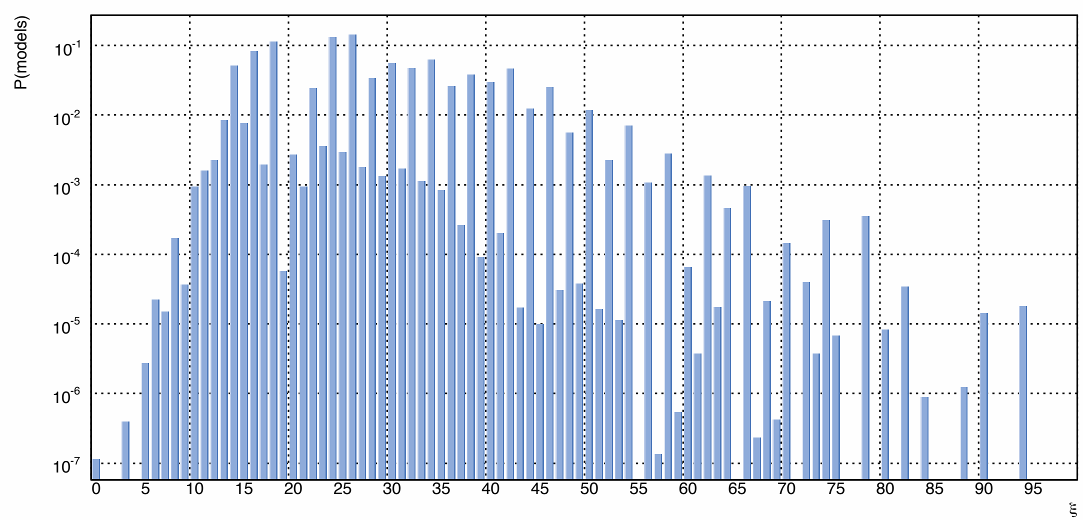

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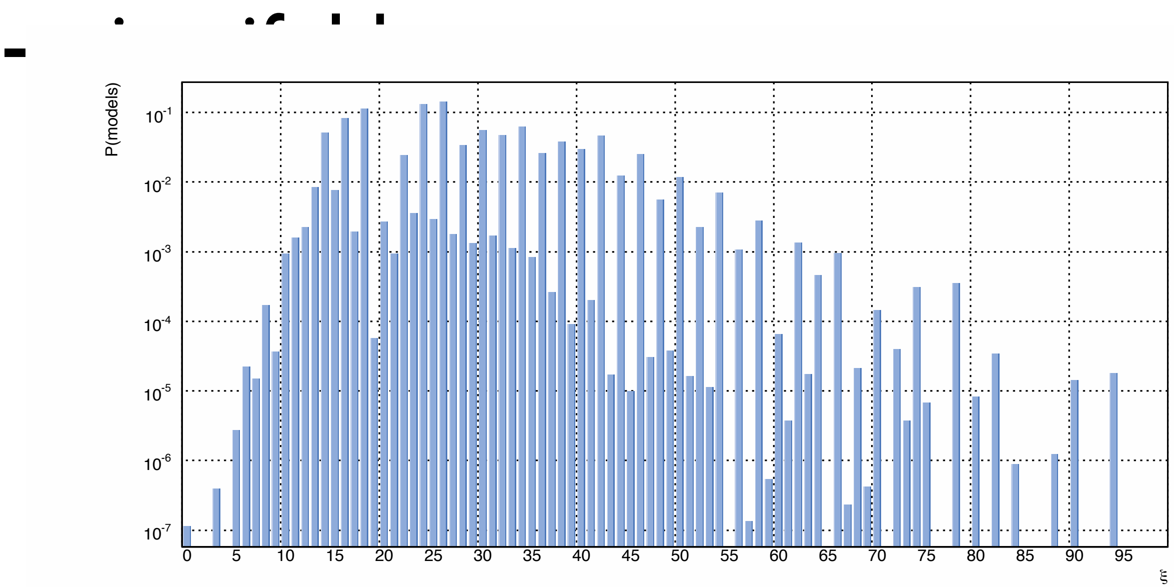

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ISB models with no chiral exotics are possible!

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Scale of wrapped D(p+3)-branes (e.g. IIB: p=0,4), (IIA: p=3):

$$(3): \quad M_p^{||} = \frac{1}{(V_p^{||})^{1/p}}, \qquad (3'): \quad M_{6-p}^{\perp} = \frac{1}{(V_{6-p}^{\perp})^{1/(6-p)}}$$

$$V_6 = V_p^{||} V_{6-p}^{\perp}$$

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(A) and (B): leave one free parameter.

M_s is a free parameter in D-brane compactifications !

There are 4 natural scenarios for the string scale:

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(o) Planck scale scenario:

M_s is the gravitational 4D Planck scale

$$\mathrm{M}_s \equiv M_{\mathrm{Planck}} \simeq 10^{19}\ \mathrm{GeV}$$

Gauge coupling unification at the Planck scales needs further effects (string threshold corrections, ...)

Alternatively relate the string scale to particles physics mass scales.

(i) GUT scale scenario:

M_s is the 4D scale of gauge coupling unification

$$\mathrm{M}_s \equiv M_{GUT} \simeq 10^{16}\ \mathrm{GeV}$$

$$\mathrm{M}_{GUT} = M_{SM} \exp\left(\frac{g_{Dp}^{-2}(M_{SM}) - g_{Dp}^{-2}(M_{GUT})}{b_p}\right)$$

Recent GUT string model building in F-theory and IIB orientifolds: (Beasly, Heckman, Marsano, Saulina, Schafer-Nameki, Vafa; Donagi, Wijnholt; Blumenhagen, Braun, Grimm, Weigand; Andreas, Curio)

- D7-branes wrapped on del Pezzo surfaces
- GUT gauge group is broken by $U(1)_Y$ flux

(ii) SUSY breaking scenario:

M_s is the intermediate 4D scale of supersymmetry breaking

(Balasubramanian, Conlon, Quevedo, Suruliz, ...)

$$\mathrm{M}_s \equiv M_{SUSY} \simeq 10^{11}\ \mathrm{GeV}$$

Gravity mediation:

$$\mathrm{M}_{SUSY} \sim \sqrt{M_{SM} M_{\mathrm{Planck}}}$$

(No natural gauge coupling unification!)

(iii) Low string scale scenario:

(Antoniadis, Arkani-Hamed, Dimopoulos, Dvali)

M_s is the Standard Model (TeV) scale:

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SUMMARY:

Table 1: The three different mass scales in D-brane models

	M_s (GeV)	L_s (m)	$M_6 = V_6^{-1/6}$ (GeV)	$V_6^{1/6}$ (m)	$M_2^\perp = (V_2^\perp)^{-1/2}$ (GeV)	$(V_2^\perp)^{1/2}$ (m)
(o)	10^{19}	10^{-35}	10^{19}	10^{-35}	10^{19}	10^{-35}
(i)	10^{16}	10^{-32}	10^{15}	10^{-31}	10^{13}	10^{-29}
(ii)	10^{11}	10^{-27}	10^{6-7}	$10^{-(22-23)}$	10^{3}	10^{-19}
(iii)	10^{3}	10^{-19}	$10^{-14/6}$	10^{-14}	10^{-13}	10^{-3}

Dimensionless volume in string units:

$$\mathrm{V}_6' = V_6 M_s^6 = \frac{M^2_{\mathrm{Planck}}}{M_s^2} = 1, 10^6, 10^{16}, 10^{32}$$

There are 3 generic type of particles:

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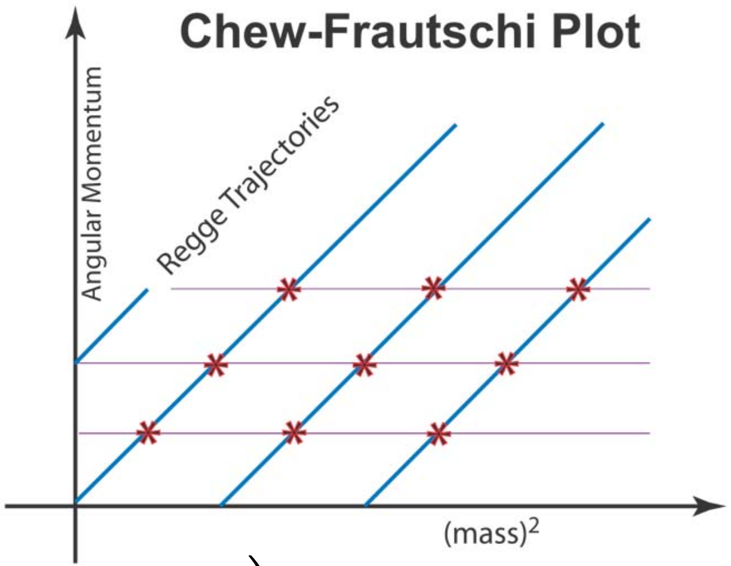
(i) Stringy Regge excitations:

$$\mathrm{M}_{\mathrm{Regge}} = M_s = \frac{M_{\mathrm{Planck}}}{\sqrt{V_6'}}$$

Open string excitations: completely universal (model independent), carry SM gauge quantum numbers



$$M_n^2 = M_s^2 \left(\sum_{k=1}^{n} \alpha_{-k}^{\mu} \alpha_k^{\nu} - 1 \right) = (n-1)\, M_s^2 \,, \quad (n = 1, \ldots, \infty)$$

(ii) Overall volume modulus:

$$M_T = \frac{M_{\text{Planck}}}{(V_6')^{3/2}} = 10^{19}, 10^{10}, 10^{-5}, 10^{-29}\ \text{GeV}$$

Closed string, model independent, neutral under the SM, interacts only gravitationally

Problem: the very light mass causes a fifth force.

Would rule out TeV string scale !

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But one expects a mass shift by radiative corrections:

$$\Delta M_T \simeq \frac{< T^\mu_\mu T^\mu_\mu >}{M^2_{\rm Planck}} \simeq \frac{M_s^4}{M^2_{\rm Planck}} \simeq 10^{-13}\ {\rm GeV}$$

(iii) D-brane cycle Kaluza Klein excitations:

$$M_{KK}^{\parallel} = \frac{1}{(V_p^{\parallel})^{1/p}} \simeq M_s = \frac{M_{\rm Planck}}{(V_6')^{1/2}}$$

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SUMMARY:

The string Regge excitations (i) and the D-brane cycle KK modes (iii) are charged under the SM and have mass of order M_s ➠ can they be seen at LHC ?!

Type IIB orientifolds: Realization of low string scale compatifications on „Swiss Cheese“ Manifolds:

(Abdussalam, Allanach, Balasubramanian, Berglund, Cicoli, Conlon, Kom, Quevedo, Suruliz;
Blumenhagen, Moster, Plauschinn;
for model building and phenomenological aspects see: Conlon, Maharana, Quevedo, arXiv:0810.5660)

Moduli potential:

Kähler potential: $K = K_{cs} - 2\log\left(V_6 + \frac{\xi}{2g_s^{\frac{3}{2}}}\right)$ (Becker, Becker, Haack, Louis)

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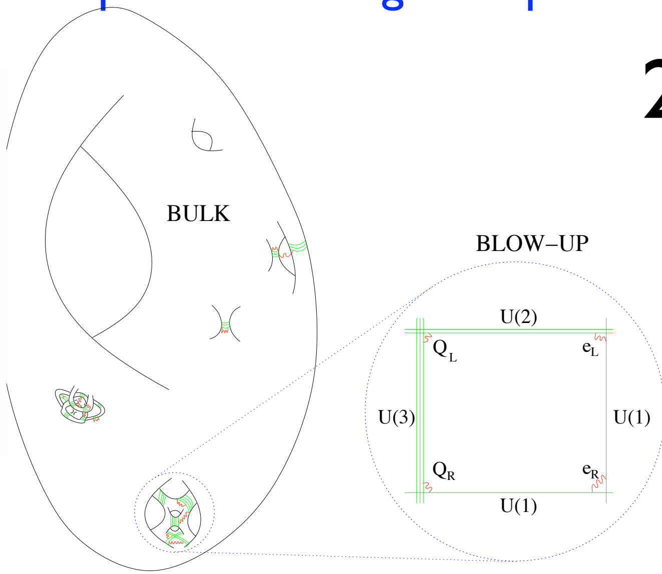
Moduli stabilization ➢

Minima: Large hierarchical scales with $V_6 M_s^6 = 10^{16}, 10^{32}$

Type IIB orientifolds: Realization of low string scale compatifications on „Swiss Cheese“ Manifolds:

(Abdussalam, Allanach, Balasubramanian, Berglund, Cicoli, Conlon, Kom, Quevedo, Suruliz;
Blumenhagen, Moster, Plauschinn;
for model building and phenomenological aspects see: Conlon, Maharana, Quevedo, arXiv:0810.5660)



2 requirements:

- Negative Euler number.
- SM lives on D7-branes around small cycles of the CY. One needs at least one blow-up mode (resolves point like singularity).

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Outline

- Intersecting D-brane models

- Mass scales in D-brane models

- Stringy amplitudes for the LHC

(The LHC string hunter's companion)

(D. Lüst, S. Stieberger, T. Taylor, arXiv:0807.3333; L. Anchordoqui, H. Goldberg, D. Lüst, S. Nawata, S. Stieberger, T. Taylor, arXiv:0808.0497 [hep-ph]; L. Anchordoqui, H. Goldberg, D. Härtl, D. Lüst, S. Nawata, O. Schlotterer, S. Stieberger, T. Taylor, to appear)

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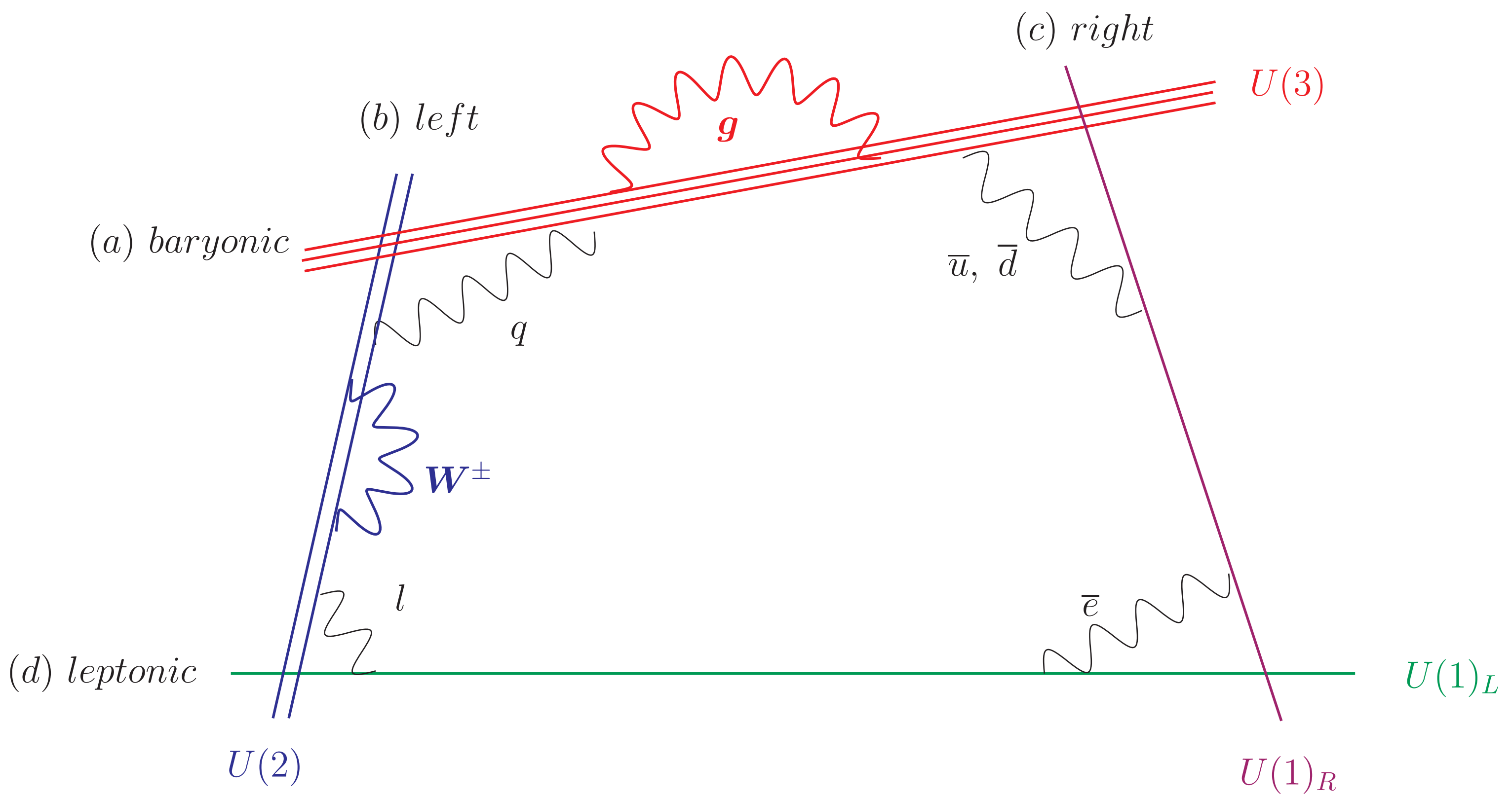
One has to compute the parton model cross sections of SM fields into new stringy states !

Parton model cross sections of SM-fields:

Disk amplitude among n external SM fields $(q, l, g, \gamma, Z^0, W^\pm)$:

$$n = 4: \quad \mathcal{A}(\Phi^1, \Phi^2, \Phi^3, \Phi^4) = < V_{\Phi^1}(z_1)\ V_{\Phi^2}(z_2)\ V_{\Phi^3}(z_3)\ V_{\Phi^4}(z_4) >_{disk}$$



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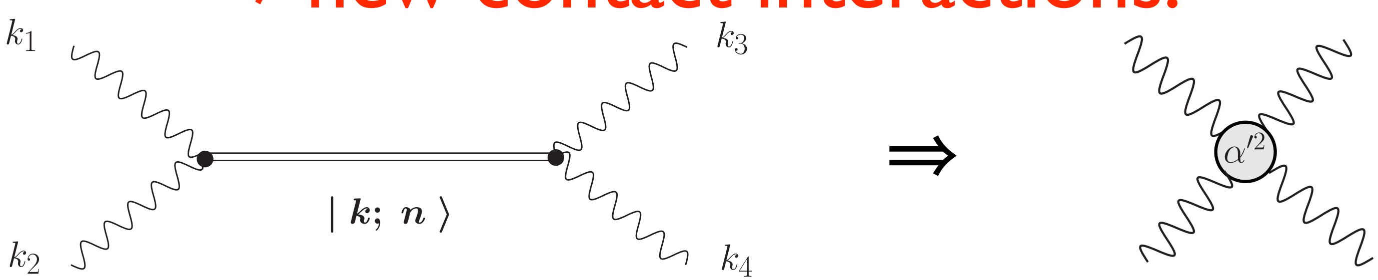
- Exchange of string Regge resonances (Veneziano like ampl.)

$\Rightarrow$ new contact interactions:



$\alpha'^2\ \zeta(2)\ \mathrm{tr} F^4$

$$\mathcal{A}(k_1, k_2, k_3, k_4; \alpha') \sim -\frac{\Gamma(-\alpha' s)\ \Gamma(1-\alpha' u)}{\Gamma(-\alpha' s - \alpha' u)} = \sum_{n=0}^{\infty} \frac{\gamma(n)}{s - M_n^2} \sim \frac{t}{s} - \frac{\pi^2}{6}\ tu\ (\alpha')^2 + \ldots$$

$$V_t(\alpha') = \frac{\Gamma(1 - s/M_{\text{string}}^2)\Gamma(1 - u/M_{\text{string}}^2)}{\Gamma(1 - t/M_{\text{string}}^2)} = 1 - \frac{\pi^2}{6} M_{\text{string}}^{-4} su - \zeta(3) M_{\text{string}}^{-6} stu + \cdots \to 1|_{\alpha' \to 0}$$

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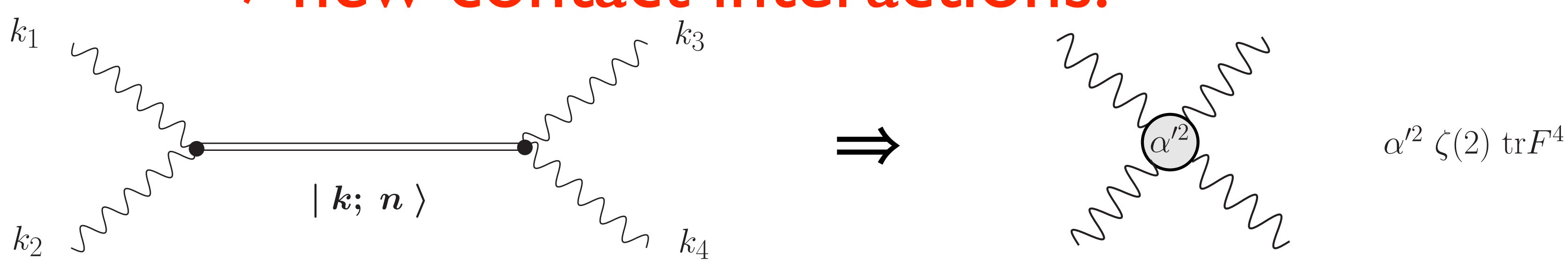

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- Exchange of KK and winding modes (model dependent)

The string scattering amplitudes exhibit some interesting properties:

- Interesting mathematical structure
- They go beyond the N=4 Yang-Mills amplitudes:

(i) The contain quarks & leptons in fundamental repr.

Quark, lepton vertex operators:

$$V_{q,l}(z,u,k) = u^{\alpha} S_{\alpha}(z) \Xi^{a\cap b}(z) e^{-\phi(z)/2} e^{ik\cdot X(z)}$$

Fermions: boundary changing (twist) operators!

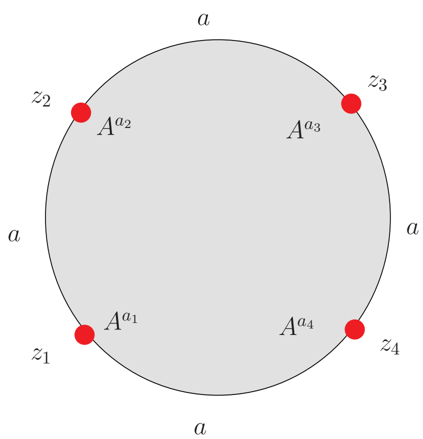
(ii) They contain stringy corrections.

Striking relation between quark and gluon amplitudes:

(i) Four point scattering amplitudes (2 jet events):

4 gluons:

(Stieberger, Taylor)



Field theory factors:

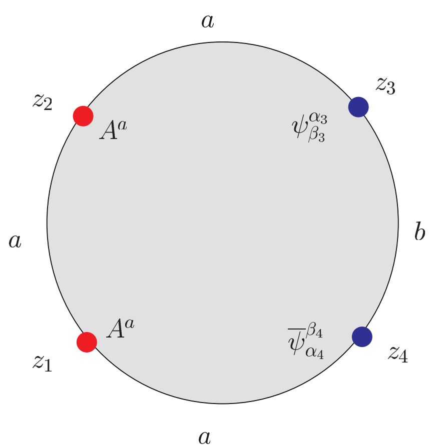
$$\mathcal{M}^{(4)}_{\rm YM} = \frac{4g^2_{\rm YM}\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$\langle ij\rangle = (\lambda_i)^\alpha(\lambda_j)_\alpha$$

$$\mathcal{A}(g_1^-, g_2^-, g_3^+, g_4^+) = V^{(4)}(\alpha', k_i) \times \mathcal{M}^{(4)}_{\rm YM}$$

2 gluons, 2 quarks:

(Lüst, Stieberger, Taylor)



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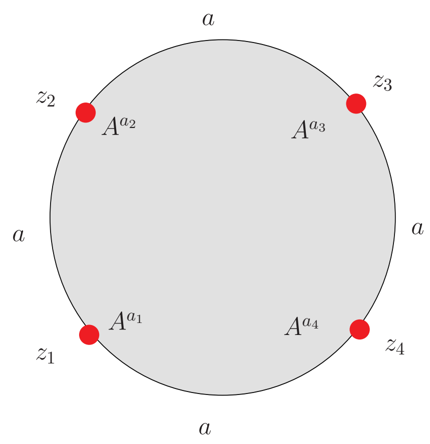

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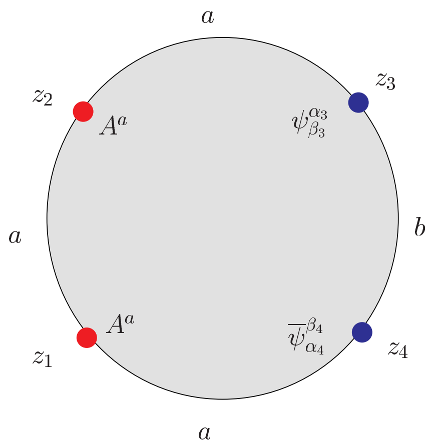

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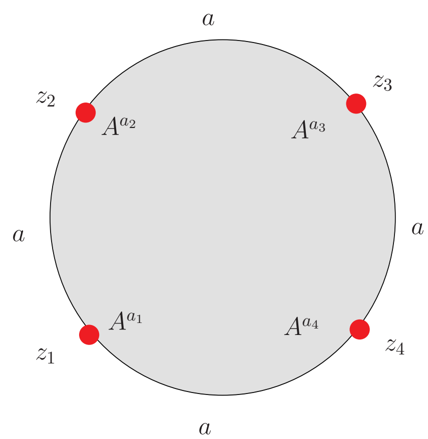

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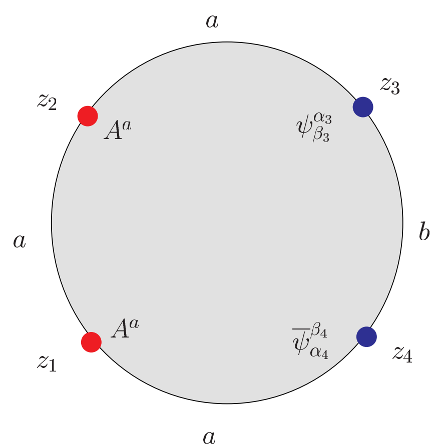

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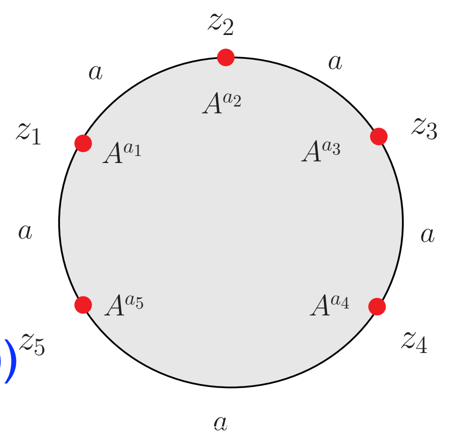
(ii) Five point scattering amplitudes (3 jet events):

5 gluons:

(Stieberger, Taylor (2006))



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$$\mathcal{M}^{(5)}_{\rm YM} = \frac{4g^3_{\rm YM}\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle 51\rangle}$$

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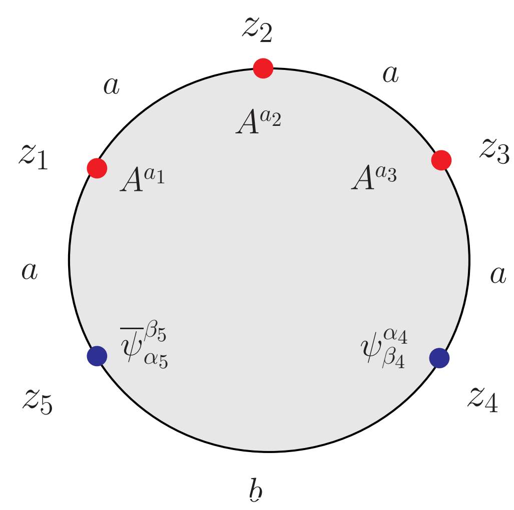
3 gluons, 2 quarks:

(D. Lüst, O. Schlotterer, S. Stieberger, T. Taylor, work in progress).



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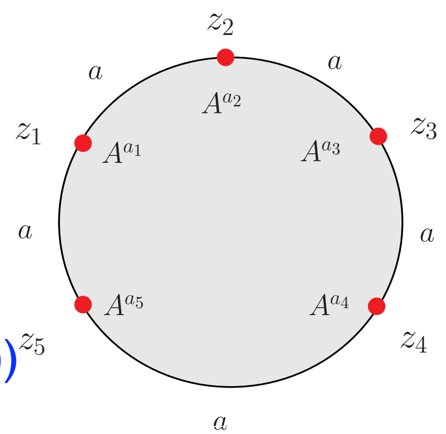

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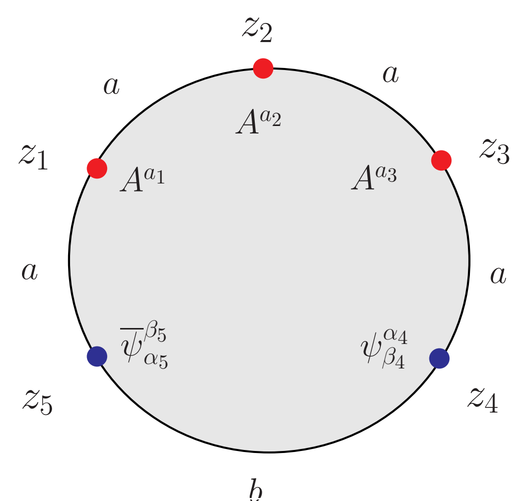

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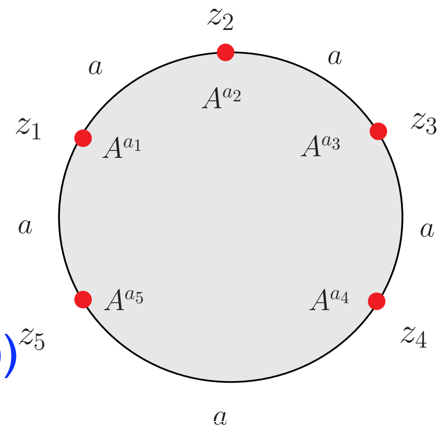

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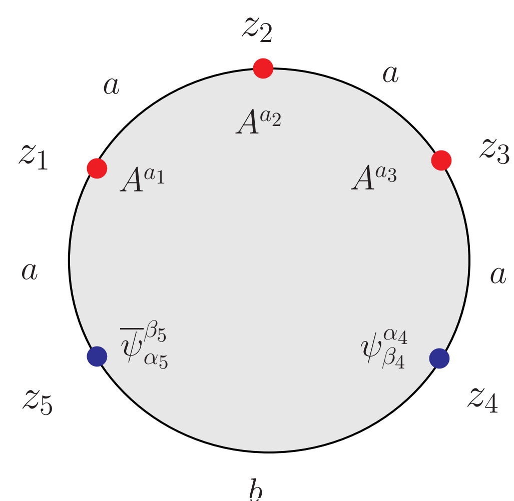

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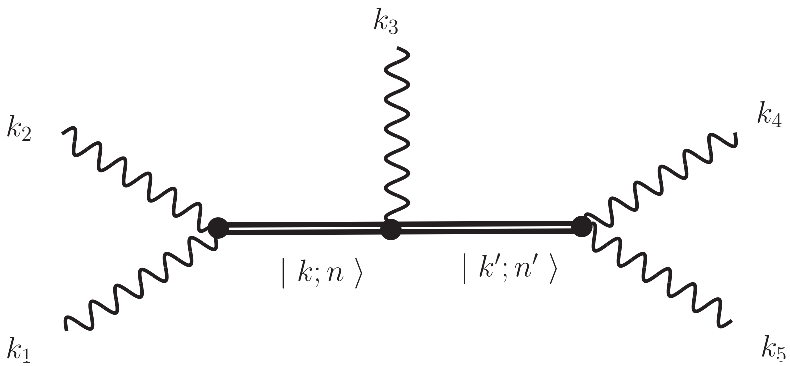
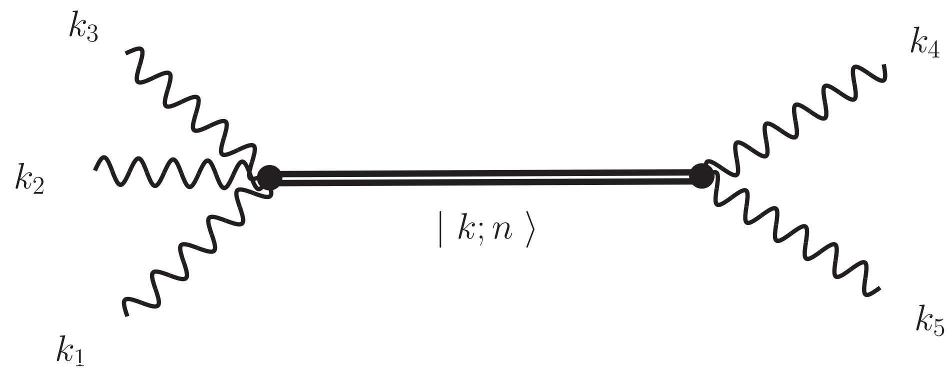
The two kinds of amplitudes are universal: the same Regge states are exchanged:



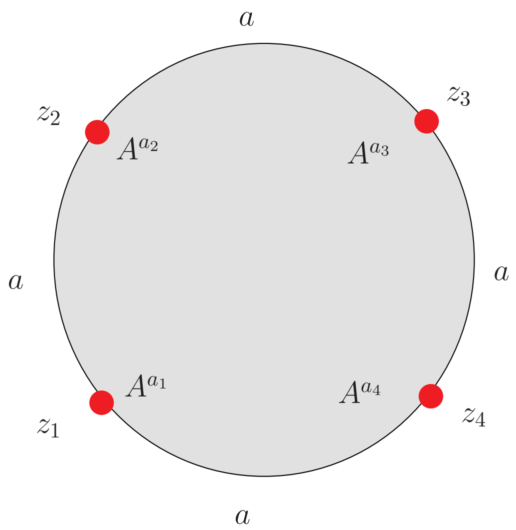


- n-point tree amplitudes with 0 or 2 open string fermions (quarks, leptons) and n or n-2 gauge bosons (gluons) are completely model independent.

$\Rightarrow$ Information about the string Regge spectrum.

4 gauge boson amplitudes:

Disk amplitude:



Only string Regge resonances are exchanged $\Rightarrow$

This amplitude is completely model independent!

Examples:

$$|\mathcal{A}(gg \to gg)|^2 = g_3^4 \left(\frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2} \right) \left[\frac{9}{4} s^2 V_s^2(\alpha') - \frac{1}{3} \left(sV_s(\alpha') \right)^2 + (s \leftrightarrow t) + (s \leftrightarrow u) \right]$$

$\Rightarrow$ dijet events

$$|\mathcal{A}(gg \to g\gamma(Z^0))|^2 = g_3^4 \frac{5}{6} Q_A^2 \left(\frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2} \right) \left(sV_s(\alpha') + tV_t(\alpha') + uV_u(\alpha') \right)^2$$

Observable at LHC for $M_{\text{string}} = 3\ \text{TeV}$

(Anchordoqui, Goldberg, Nawata, Taylor, arXiv:0712.0386)

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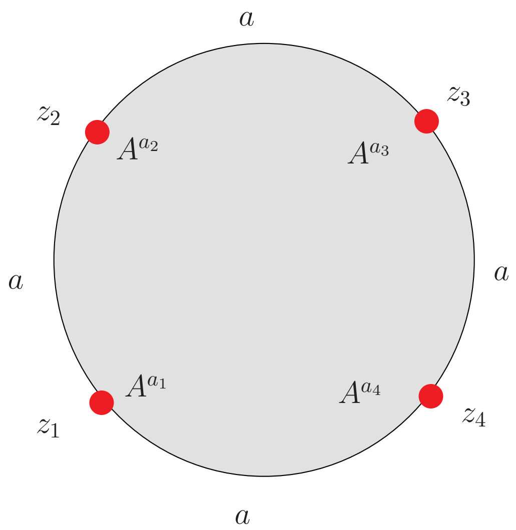

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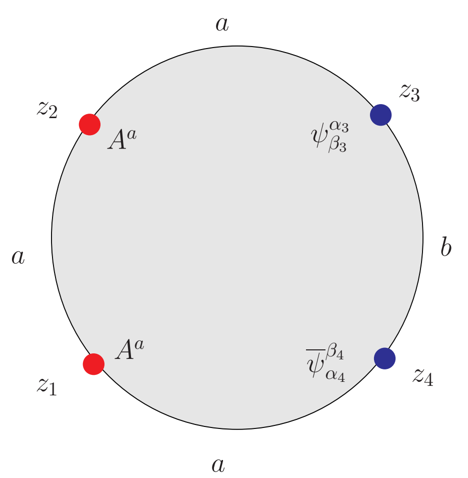
Examples:

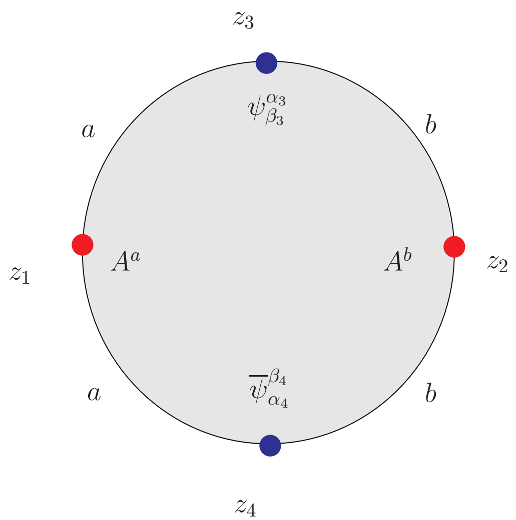
$\alpha' \longrightarrow 0$: agreement with SM!

$$|\mathcal{A}(gg \to gg)|^2_{\alpha' \to 0} \to \Big(\frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2}\Big)\frac{9}{4}\big(s^2 + t^2 + u^2\big)$$

$$|\mathcal{A}(gg \to \gamma(Z^0))|^2_{\alpha' \to 0} \to 0$$

2 gauge boson - two fermion amplitude:





Note: Cullen, Perelstein, Peskin (2000) considered: $e^+e^- \to \gamma\gamma$

Only string Regge resonances are exchanged $\Rightarrow$

These amplitudes are completely model independent!

$$|\mathcal{A}(qg \to qg)|^2 = g_3^4 \frac{s^2+u^2}{t^2}\left[V_s(\alpha')V_u(\alpha') - \frac{4}{9}\frac{1}{su}(sV_s(\alpha') + uV_u(\alpha'))^2\right]$$

$\Rightarrow$ dijet events

$$|\mathcal{A}(qg \to q\gamma(Z^0))|^2 = -\frac{1}{3}g_3^4 Q_A^2 \frac{s^2+u^2}{sut^2}(sV_s(\alpha') + uV_u(\alpha'))^2$$

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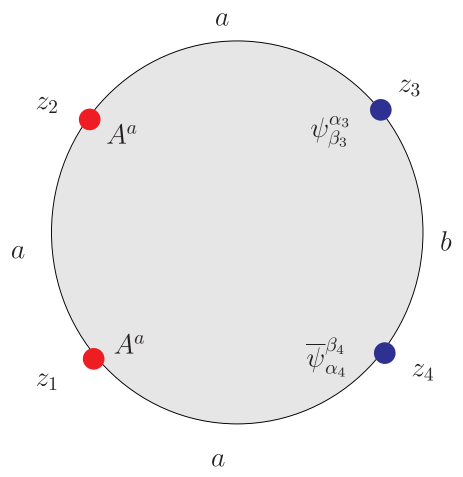

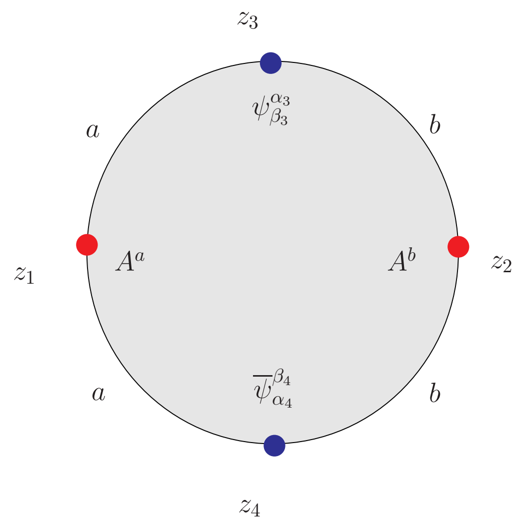

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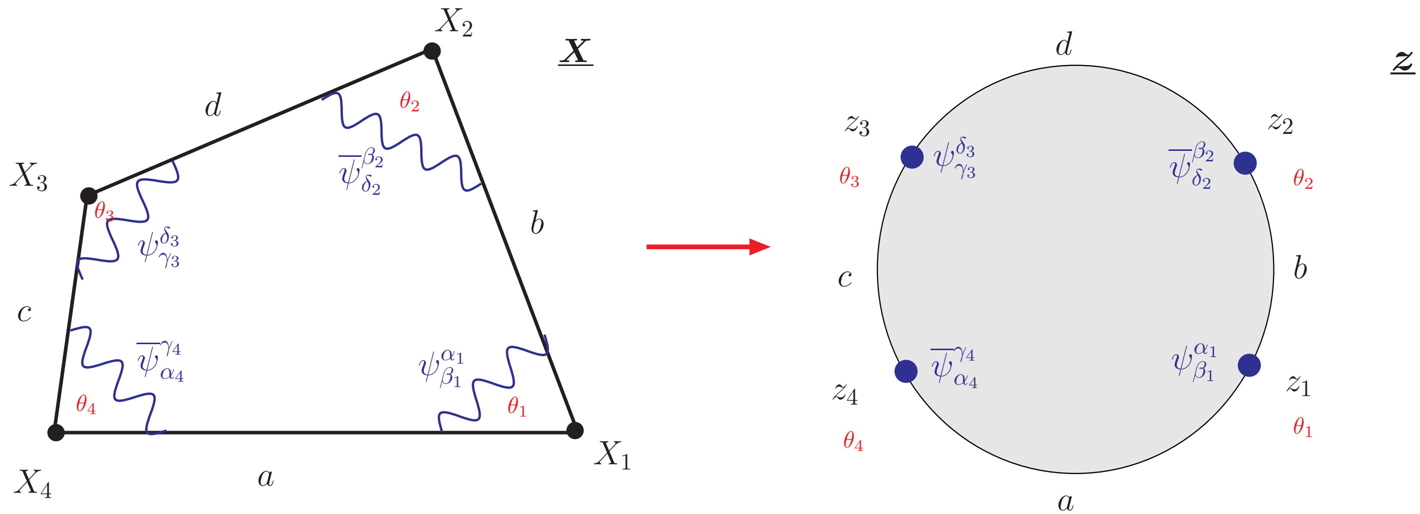
$$|\mathcal{A}(qg \rightarrow q\gamma(Z^0))|^2_{\alpha' \rightarrow 0} = -\frac{1}{3} g_3^4 Q_A^2 \frac{s^2+u^2}{sut^2}(s+u)^2$$

4 fermion amplitudes:



Exchange of Regge, KK and winding resonances.

These amplitudes are more model dependent and test the internal CY geometry.

Constrained by FCNC's and/or proton decay.

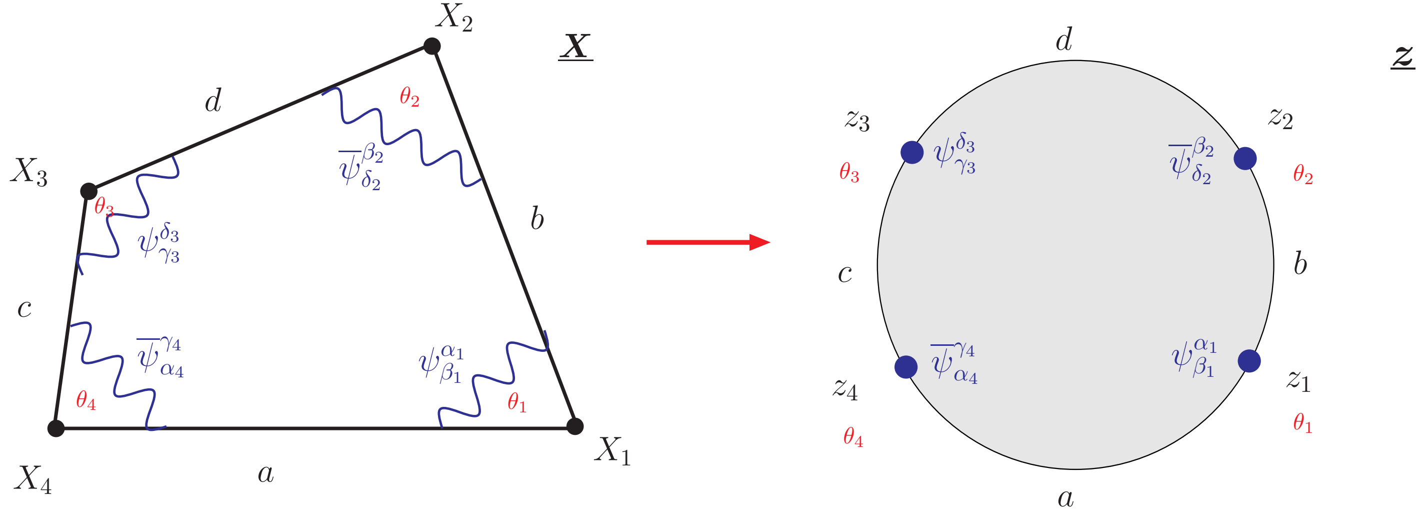
(Klebanov, Witten, hep-th/0304079; Abel, Lebedev, Santiago, hep-th/0312157)

E.g.

$$|\mathcal{A}(qq \to qq)|^2 = \frac{2}{9}\frac{1}{t^2}\Big[\big(sF^{bb}_{tu}(\alpha')\big)^2 + \big(sF^{cc}_{tu}(\alpha')\big)^2 + \big(uG^{bc}_{ts}(\alpha')\big)^2 + \big(uG^{cb}_{ts}(\alpha')\big)^2\Big] + \frac{2}{9}\frac{1}{u^2}\Big[\big(sF^{bb}_{ut}(\alpha')\big)^2 + \big(sF^{cc}_{ut}(\alpha')\big)^2 + \big(tG^{bc}_{us}(\alpha')\big)^2 + \big(tG^{cb}_{us}(\alpha')\big)^2\Big] - \frac{4}{27}\frac{s^2}{tu}F^{bb}_{tu}(\alpha')F^{bb}_{ut}(\alpha') + F^{cc}_{tu}(\alpha')F^{cc}_{ut}(\alpha'))$$

depend on internal geometry

4 fermion amplitudes:



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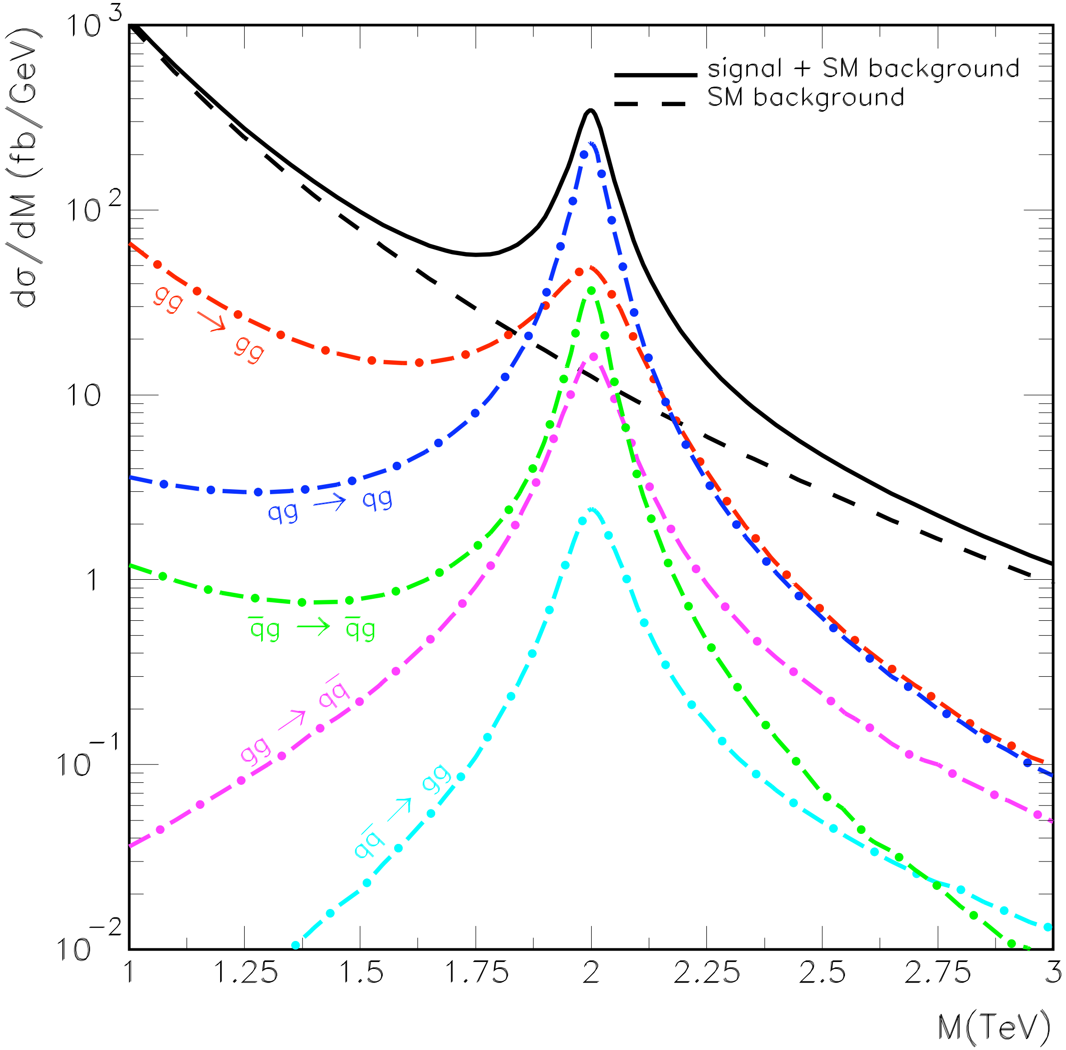
E.g.

$\alpha' \to 0$: agreement with SM !

$$|\mathcal{A}(qq \to qq)|^2_{\alpha' \to 0} \to \frac{4}{9}\left[\frac{s^2+u^2}{t^2}\right] + \frac{4}{9}\left[\frac{s^2+t^2}{u^2}\right] - \frac{8}{27}\frac{s^2}{tu}$$

These stringy corrections can be seen in dijet events at LHC:



(Anchordoqui, Goldberg, Lüst, Nawata, Stieberger, Taylor, arXiv:0808.0497[hep-ph])

$$M_{\rm Regge} = 2 \text{ TeV}$$

$$\Gamma_{\rm Regge} = 15 - 150 \text{ GeV}$$

Widths can be computed in a model independent way !

(Anchordoqui, Goldberg, Taylor, arXiv:0806.3420)

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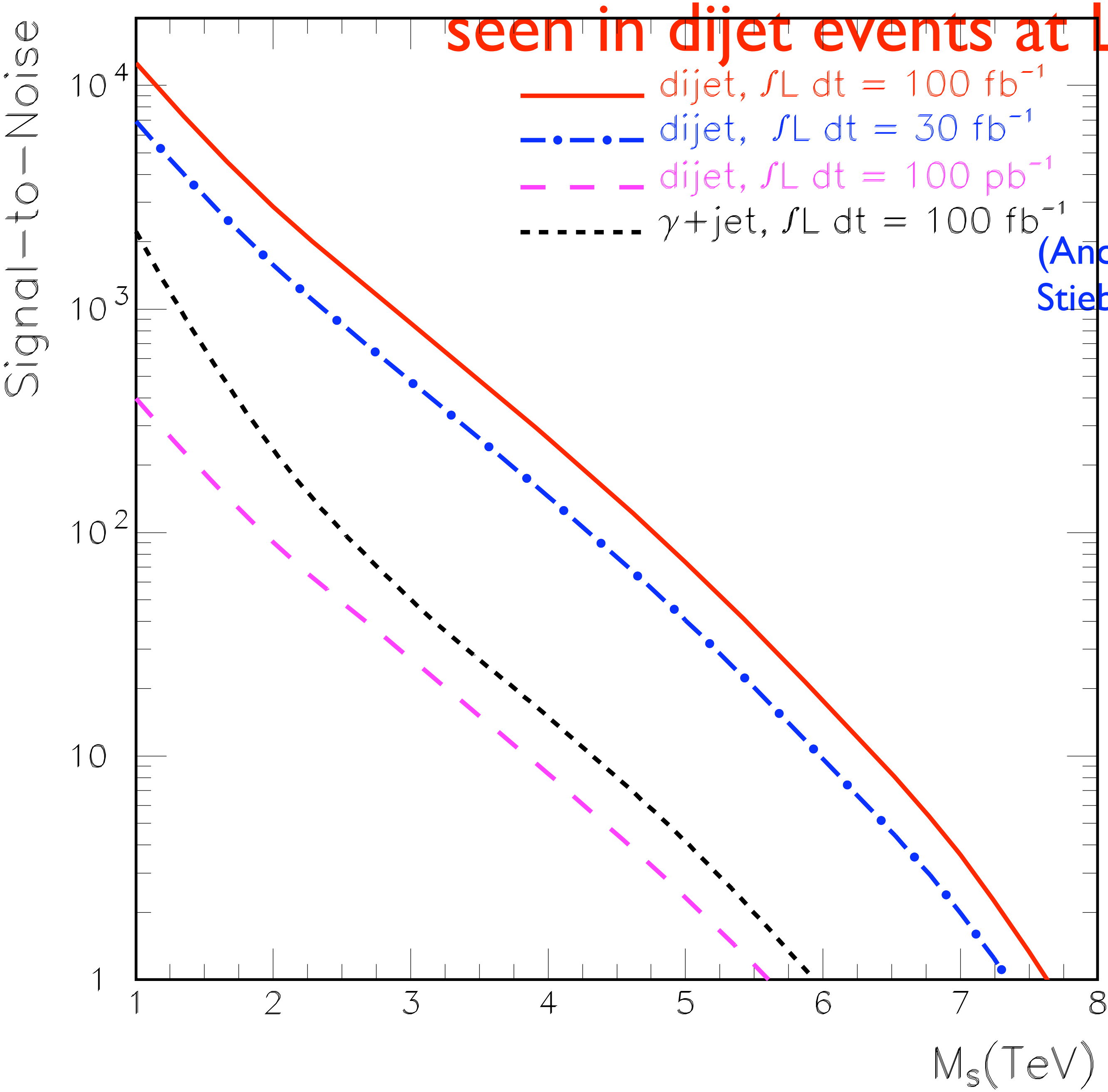

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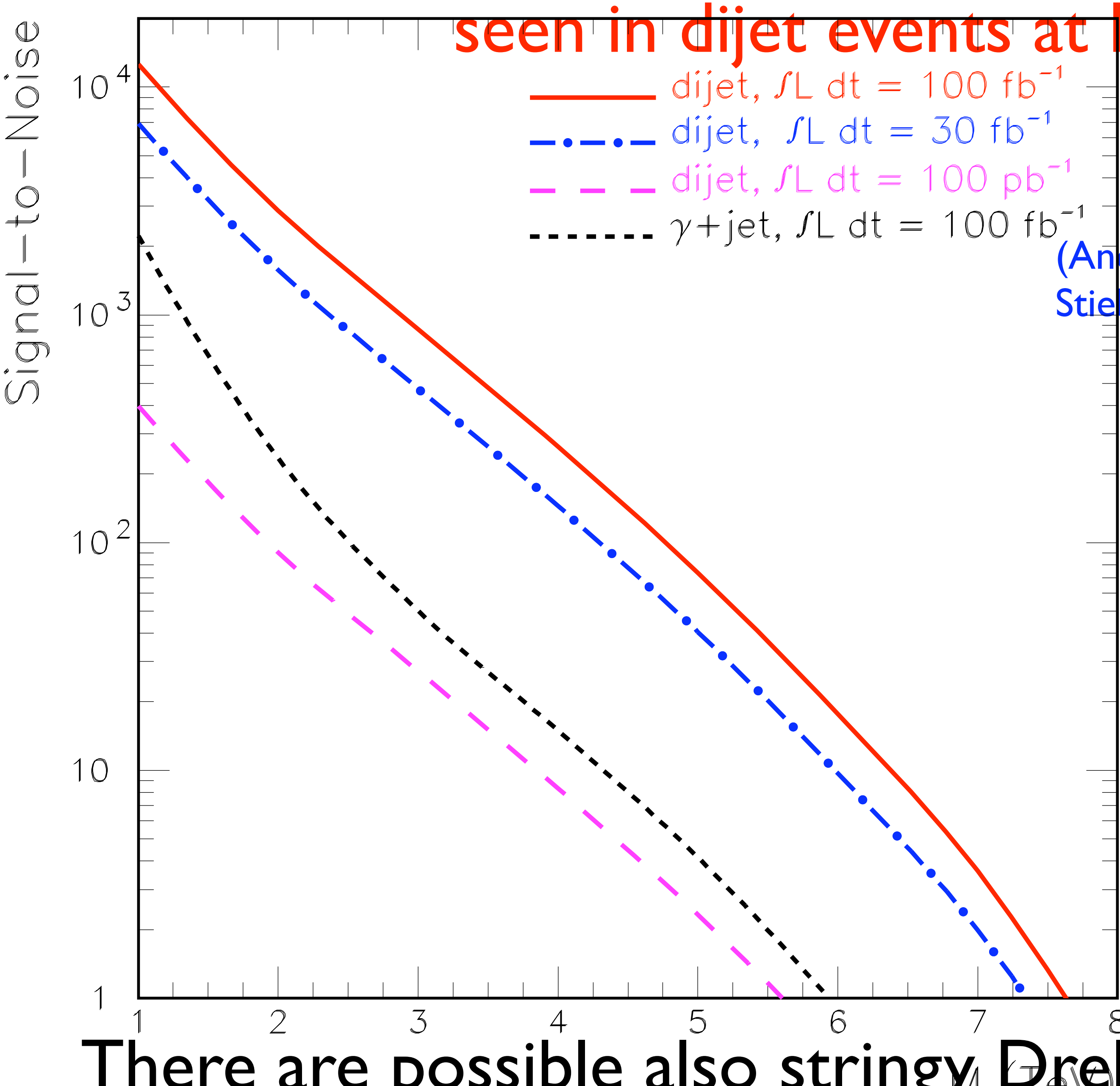

(Anchordoqui, Goldberg, Lüst, Nawata, Stieberger, Taylor, arXiv:0808.0497[hep-ph])

There are possible also stringy Drell-Yan processes like

$$q\bar{q} \longrightarrow l\bar{l}$$

- KK modes are seen in scattering processes with more than 2 fermions.

$\Rightarrow$ Information about the internal geometry.

KK modes are exchanged in t- and u-channel processes and exhibit an interesting angular distribution.

(L. Anchordoqui, H. Goldberg, D. Lüst, S. Nawata, S. Stieberger, T. Taylor, paper in preparation)

Conclusions

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(Independent of amount of (unbroken) supersymmetry!)

String tree level, 4-point processes with 2 or 4 gluons

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Black holes are heavier than Regge states: $M_{b.h.} = \frac{M_{\rm string}}{g_{\rm string}}$

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Question: do loop and non-perturbative corrections change tree level signatures? Onset of n.p. physics: $M_{b.h.}$

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- On … tions:

(Inde … metry!)

Strin … ons

??

Compu … Black … $\frac{M_{\text{string}}}{g_{\text{string}}}$

INTERESTING TIMES FOR STRING PHENOMENOLOGY ARE AHEAD OF US.

THANK YOU !!

Question: do loop and non-perturbative corrections change tree level signatures? Onset of n.p. physics: $M_{b.h.}$

Any null-vector $k_i^2 = 0$ can be written in terms of two spinors $(\lambda, \tilde{\lambda})$

Momentum $k_i^\mu \longrightarrow$ Dirac spinor $\begin{pmatrix} u_+(k_i)_\alpha \\ u_-(k_i)_{\dot{\alpha}} \end{pmatrix} \equiv \begin{pmatrix} (\lambda_i)_\alpha \\ (\tilde{\lambda}_i)_{\dot{\alpha}} \end{pmatrix}$

$$u(k) = \text{Dirac spinor, helicity states } u_\pm(k) = (1 \pm \gamma_5)\, u(k)$$

with choice $u_+(k) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{k^+} \\ \sqrt{k^-}\, e^{i\varphi} \\ \sqrt{k^+} \\ \sqrt{k^-}\, e^{i\varphi} \end{pmatrix}$, $u_-(k) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{k^-}\, e^{-i\varphi} \\ -\sqrt{k^+} \\ \sqrt{k^-}\, e^{-i\varphi} \\ \sqrt{k^+} \end{pmatrix}$

$$k^\pm = k^0 \pm k^3$$

$$e^{\pm i\varphi} = \frac{k^1 \pm ik^2}{\sqrt{k^+ k^-}}$$

Define $\boxed{|\, i^\pm \rangle = u_\pm(k_i) \quad , \quad \langle i^\pm \,| = \overline{u_\pm(k_i)}}$

Spinor products:

$$\langle ij \rangle := \langle i^- | j^+ \rangle = \overline{u_-(k_i)}\; u_+(k_j) \equiv \epsilon^{\alpha\beta}\; (\lambda_i)_\alpha\; (\lambda_j)_\beta = \sqrt{k_i k_j}\; e^{i\phi_{ij}} \;,$$
$$[ij] := \langle i^+ | j^- \rangle = \overline{u_+(k_i)}\; u_-(k_j) \equiv \epsilon^{\dot{\alpha}\dot{\beta}}\; (\tilde{\lambda}_i)_{\dot{\alpha}}\; (\tilde{\lambda}_j)_{\dot{\beta}} = -\sqrt{k_i k_j}\; e^{-i\phi_{ij}}$$

$$\Longrightarrow \quad \boxed{\langle ij \rangle [ji] = -k_i k_j}$$