Three-dimensional Newtonian photorealistic ray tracing of the conformal grating cloak

Jad C. Halimeh[1], Roman Schmied[2], and Martin Wegener[3]
[1] *Physics Department, Arnold Sommerfeld Center for Theoretical Physics, and Center for NanoScience, Ludwig-Maximilians-Universität München, D-80333 München, Germany*
[2] *Departement Physik, Universität Basel, Switzerland*
[3] *Institut für Angewandte Physik, DFG-Center for Functional Nanostructures (CFN), and Institut für Nanotechnologie, Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany*
martin.wegener@kit.edu

Abstract: Conformal grating cloaks are relatives of the carpet cloak and can deliver excellent cloaking for reasonable parameters. This is illustrated by photorealistic Newtonian ray-tracing calculations and by cross-correlation coefficients based on these.
OCIS codes: (230.3205) Invisibility cloaks; (160.3918) Metamaterials; (080.2710) Inhomogeneous optical media.

1. Introduction

Practical invisibility cloaking structures have recently come within reach *via* the concept of transformation optics [1]. The broadband dielectric carpet-cloak design introduced by Jensen Li and John Pendry [2] immediately stimulated several experimental studies [3-6]. In the carpet cloak, an arbitrarily shaped object is hidden underneath a metallic carpet (or "ground plane"). The resulting bump in the mirror profile can be made to appear flat, hence invisible, by adding a tailored graded-refractive-index structure on top. However, the finite-size quasi-conformal carpet cloak [2] (that is numerically evaluated by minimizing the modified-Liao functional) suffers from an inherent limitation, namely a pronounced beam displacement that was recently emphasized [7].

More recently, we have introduced an analytical and strictly conformal version of the carpet cloak [8]. We have shown that mirror profiles with zero average value lead to a much more rapid decay of the refractive-index profile away from the bump. This has essentially eliminated the undesired beam displacement and has made even finite-size cloaking structures almost perfect in two dimensions. The grating cloak [8] is just one example out of this new class of two-dimensional cloaks. However, its cloaking performance in three dimensions has neither been directly visualized nor has it been quantified. Both aspects are addressed in this previously unpublished work.

2. Computational Results

In two dimensions, conformal maps lead to perfect cloaking in both ray optics and wave optics. Regarding carpet cloaks, we have recently introduced [8] a new class of conformal maps parameterized as

$$z \to f(z) = z + \int_0^\infty c_k e^{ikz} dk \qquad (1)$$

The real and imaginary parts of $f(z)$ and of z correspond to the two spatial coordinates of the real and virtual space, respectively. The grating cloak results from the special case where the coefficients c_k are only non-zero for a single non-zero spatial frequency k, for which $c_k=c$. The refractive-index profile, n, results from

$$n(f(z)) = \left|\frac{1}{df/dz}\right| = \frac{1}{|1+W_0(icke^{ikf(z)})|}, \qquad (2)$$

with the Lambert function (or product logarithm) W_0.

This index distribution (2) will cloak a mirror profile that is generally given by a trochoid, which in the shallow-bump limit reduces to a simple cosine function. We have shown [8] that the aforementioned beam displacement [7] of the quasi-conformal carpet cloak is practically negligible for the finite-size conformal grating cloak [8].

To visualize the grating cloak's performance in three dimensions, we have performed ray-tracing calculations. We have found that a Newtonian approach is numerically advantageous compared to using Snell's law. In the Newtonian approach (that is closely related to a Hamiltonian approach), we solve the second-order differential equation

$$\frac{d^2\vec{r}}{dt^2} = \frac{|\vec{v}|^2 \vec{\nabla} n - 2(\vec{\nabla} n \cdot \vec{v})\vec{v}}{n} \tag{3}$$

for the coordinate vector *r* of the light ray, which is influenced by the gradient of the refractive-index profile (2) and the local phase velocity of light *v*. A set of examples of rendered images is illustrated in Fig. 1. A model stands in front of a mirror and observes the depicted images. The vertical (horizontal) field of view is FOV=40° (FOV=50°), which is similar to the human *focal* FOV.

To quantify the cloaking performance, we have calculated the cross-correlation coefficients *C* of the various images. For example, for small horizontal FOV and for Fig. 1(a) and Fig. 1(c), we find *C*=99%. Absolutely perfect cloaking would correspond to *C*=100%. For a horizontal FOV=10°, we still get *C*=90%. Corresponding calculations have also been performed for various "referenced cases" embedded in air, in which the refractive index profile does *not* exhibit values below unity like in (2) and in Fig. 1, but is rather restricted to the experimentally easily accessible interval [1.0,2.2]. These results are comparable to those in Fig. 1 (not depicted).

Fig. 1. Images rendered by Newtonian ray tracing. A model looks at herself in a flat mirror (left), in a grating mirror (middle) leading to pronounced distortions, and in a grating mirror with grating cloak on top (right). The distance between the model and the mirror is 50 cm, the peak-to-peak grating amplitude is 2.2 cm, the grating period is $2\pi/k$=18.2 cm, and the height of the grating cloak is 20.8 cm. Note that the cloaking performance is close to perfect in the middle of the image (compare left and right), whereas deviations occur on the left and on the right-hand side of the image on the right (bent bookshelves).

3. References

[1] M. Wegener and S. Linden, "Shaping Optical Space with Metamaterials," Physics Today **63**, 32-36 (2010).

[2] J. Li and J. B. Pendry, "Hiding under the carpet: a new strategy for cloaking," Phys. Rev. Lett. **101**, 203901 (2008).

[3] R. Liu, C. Ji, J. J. Mock, J. Y. Chin, T. J. Cui, and D. R. Smith, "Broadband ground-plane cloak," Science **323**, 366-369 (2009).

[4] J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, "An optical cloak made of dielectrics," Nature Mater. **8**, 568-571 (2009).

[5] L. H. Gabrielli, J. Cardenas, C. B. Poitras, and M. Lipson, "Cloaking at optical frequencies," Nature Photon. **3**, 461-463 (2009).

[6] T. Ergin, N. Stenger, P. Brenner, J. B. Pendry, and M. Wegener, "Three-Dimensional Invisibility Cloak at Optical Wavelengths," Science **328**, 337-339 (2010).

[7] B. Zhang, T. Chan, and B.-I. Wu, "Lateral Shift Makes a Ground-Plane Cloak Detectable," Phys. Rev. Lett. **104**, 233903 (2010).

[8] R. Schmied, J. C. Halimeh, and M. Wegener, "Conformal carpet and grating cloaks," Opt. Express **18**, 24361-24367 (2010).