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Theoretical investigation of the electron velocity in quantum Hall bars, in the out of linear response regime

A. Siddiki[a], D. Eksi[b],*, E. Cicek[b], A.I. Mese[b], S. Aktas[b], T. Hakioğlu[c]

[a]*Physics Department, Arnold Sommerfeld Center for Theoretical Physics, and Center for NanoScience, Ludwig-Maximilians-Universität München, D-80333 Munich, Germany*
[b]*Department of Physics, Trakya University, 22030 Edirne, Turkey*
[c]*Department of Physics and UNAM Material Science and Nanotechnology Research Institute, Bilkent University, Ankara 06800, Turkey*

Abstract

We report on our theoretical investigation of the electron velocity in (narrow) quantum Hall systems, considering the out-of-linear-response regime. The electrostatic properties of the electron system are obtained by the Thomas–Fermi–Poisson nonlinear screening theory. The electron velocity distribution as a function of the lateral coordinate is obtained from the slope of the screened potential within the incompressible strips (ISs). The asymmetry induced by the imposed current on the ISs is investigated, as a function of the current intensity and impurity concentration. We find that the width of the IS on one side of the sample increases linearly with the intensity of the applied current and decreases with the impurity concentration.
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In the conventional models of the quantum Hall effect (QHE), the Coulomb interaction is ignored and either localization or the 1D edge states (ESs) was accepted as the explanation. However, recent experimental [1] and theoretical [2] investigations of the two-dimensional electron systems (2DESs) under strong perpendicular magnetic fields B provided information about the local quantities, such as potential, compressibility, current and electron density. The findings point out clearly the importance of the involved interactions at narrow ($\lesssim 10\,\mu m$) samples, which manifest themselves by the formation of the compressible and incompressible regions. More recently, the experiments performed in the integer QHE regime [3] promote the possibility of inferring interaction mechanisms between the ESs, using an electronic version of Mach–Zehnder interferometer. The surprising results, up to now, cannot be explained within the naive single particle pictures, where the group velocity of the electrons (v_{el}) is

assumed to be constant and the imposed current is believed to be carried by the Büttiker type ESs.

Recently, we have investigated v_{el} depending on the sample properties, including the Coulomb interaction [4]. We used the screening model to calculate the electron and potential distribution and obtained v_{el} from the slope of the screened potential at the Fermi level and also across the incompressible strips (ISs), where the current flows from. The calculations were done at equilibrium and within the linear response regime. We found that v_{el} strongly depends on the sample properties, in the case of small currents. Here, we extend our investigation to a regime, where the imposed external current I is high enough to change both the electron and potential distribution, i.e., out-of-linear-response (OLR) regime. The effect of the current is included self-consistently by employing the scheme developed by Güven and Gerhardts [2].

Here, we confine ourselves to the historical Chklovskii model geometry [5]. We assume that electrons and donors are on the same xy plane ($z = 0$) and donors are distributed homogeneously in the interval $-d < x < d$, where $2d$ is the

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*Corresponding author.

E-mail address: denizeksi@gmail.com (D. Eksi).

sample width. On the other hand electrons are depleted from the edges and the translational invariant electron channel is formed in the interval $-b < x < b$, where $|d| > |b|$ and $|d - b|$ is called the depletion length. We consider spinless electrons by setting the spin degeneracy to two, $g_s = 2$; therefore ISs will assume only even local filling factors, $v(x)$. First we calculate the electron density and the screened potential from the following self-consistent (SC) equations:

$$n_{\text{el}}(x) = \int dE \frac{D(E)}{e^{[E+V(x)-\mu^\star]/k_B T} + 1} \tag{1}$$

and

$$V(x) = -\frac{2e^2}{\bar{\kappa}} \int_{-d}^{d} dx' K(x, x')(n_0 - n_{\text{el}}(x')), \tag{2}$$

within the Thomas–Fermi approximation (TFA), which assumes that the electrostatic quantities vary slowly in the quantum mechanical scales, such as the magnetic length $l = \sqrt{\hbar/m\omega_c}$ ($\omega_c = eB/mc$). Eq. (1) describes $n_{\text{el}}(x)$ as a function of the electrochemical potential $\mu^\star$ (which is constant in the absence of I), temperature (T) and total potential energy $V(x)$, where $D(E)$ is the Gaussian broadened density of states (DOS) given by

$$D(E) = \frac{1}{2\pi l^2} \sum_{n=0}^{\infty} \frac{\exp(-[E_n - E]^2/\Gamma^2)}{\sqrt{\pi}\Gamma}, \tag{3}$$

with the impurity parameter Γ, which gives the Landau level (LL) broadening and the Landau energy $E_n = \hbar\omega_c(n + \frac{1}{2})$, whereas Eq. (2) relates the charge distribution with the total potential. We keep the donor distribution fixed, with a constant surface number density n_0, and obtain $n_{\text{el}}(x)$ iteratively. Here $\bar{\kappa}$ is an average dielectric constant and $K(x, x')$ is the solution of the Poisson equation preserving the boundary conditions, $V(-d) = V(d) = 0$. The confinement potential can be calculated analytically yielding, $V_{\text{conf}}(x) = E_0\sqrt{1 - (x/d)^2}$, where $E_0 = (2\pi e^2/\bar{\kappa})n_0 d$ is the pinch-off energy. In thermal equilibrium we solve these two equations iteratively, keeping the average electron density constant, starting from $T = 0$ and $B = 0$ solutions.

In the presence of an external (fixed) I driven in longitudinal direction, the situation is fairly different. Since the imposed current induces modifications on $n_{\text{el}}(x)$ and $V(x)$, one should include this effect self-consistently into the above scheme, which is done by assuming a local thermal equilibrium. The driving electric field is given by the gradient of the (now, position-dependent) electrochemical potential, $E(\mathbf{r}) = \nabla\mu^\star(\mathbf{r})/e = \hat{\rho}(\mathbf{r})j(\mathbf{r})$. With translational invariance and keeping the intensity of I fixed, one can obtain the current distribution and the position-dependent electrochemical potential, for a given local resistivity $(\hat{\rho}(\mathbf{r}) = [\sigma(n_{\text{el}}(x))]^{-1})$ tensor. In the next step this $\mu^\star(x)$ will be used to obtain the new $n_{\text{el}}(x)$ and $V(x)$ in a second iteration loop [2,6].

We first investigate the effect of the current intensity $I = (U_H/e)[d/b](e^2\bar{n}_{\text{el}}/m\omega_c)E_0$, measured in units of

$\hbar\omega_c(= \Omega)$ on the widths (W_2) of the ISs with $v(x) = 2$. For a detailed discussion of the electron and potential distribution, we suggest the reader to check Fig. 1 and the related text of Ref. [2]. In Fig. 1, we show the evolution of the IS widths as a function of the current intensity for three B values. We see that W_2 on both sides are (almost) linearly dependent on I, for the right IS it increases and for the left IS decreases. We observe that the total width of the IS increases by increasing the intensity. For a given magnetic field, the average electron velocity is defined as $v_y(x) = (1/\hbar)(\partial V(x)/\partial x)$, within the TFA imposing $E_n(X) = E_n + V(X)$ and the center coordinate can be replaced by the spatial coordinate x. If one calculates the v_{el} at the right IS, it will decrease by increasing I, since at a fixed B, the height of the potential drop remains constant, namely Ω, meanwhile the thickness of the IS increases. We should also note that, although the velocity of the electrons decreases, the number of electrons increases due to a wider IS, therefore more current is carried at the right IS over all. In Fig. 2, we plot the effect of DOS broadening on the IS widths, again for three selected B field values and current intensities. It is known that if the widths of the ISs become small or comparable with the magnetic length they essentially disappear [2]; however, here we still observe them as an artifact of TFA. We see that the left ISs become narrower than the magnetic length compared to the right ISs, due to the strong current-induced-asymmetry. We observe that the impurity concentration affects the W_2 in a non-linear manner strongly and if the DOS broadening exceeds 20% of the cyclotron energy, no ISs are left. At zero bias the transition between having an IS or not is rather smooth, meanwhile this transition occurs much drastically in the case of $I \neq 0$.
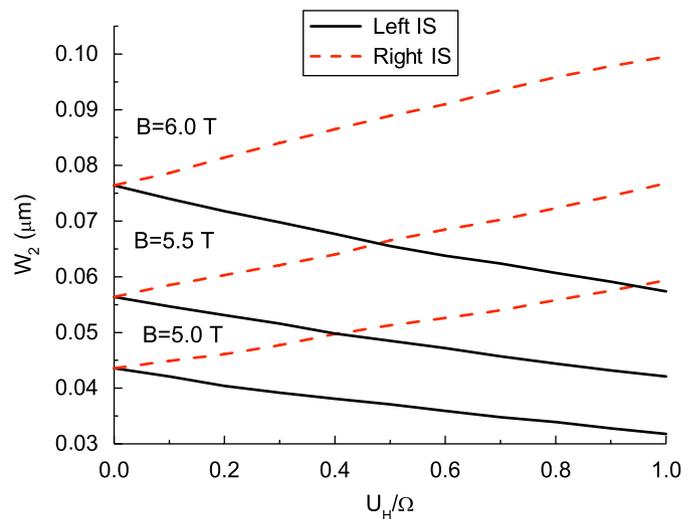

Fig. 1. The widths of the ISs at two edges of a narrow sample ($2d = 2\,\mu m$) for three typical B values at $T = 1\,K$ vs. I. The widths of the ISs increase (almost) linearly at the right edge (broken lines), whereas decreases similarly on the opposite edge. The impurity concentration is chosen such that the corresponding DOS broadening $\gamma = \Gamma/\hbar\omega_c = 0.025$. The number density of the donors is fixed $n_0 = 4 \times 10^{11}\,cm^{-2}$, whereas the depletion length is 200 nm.
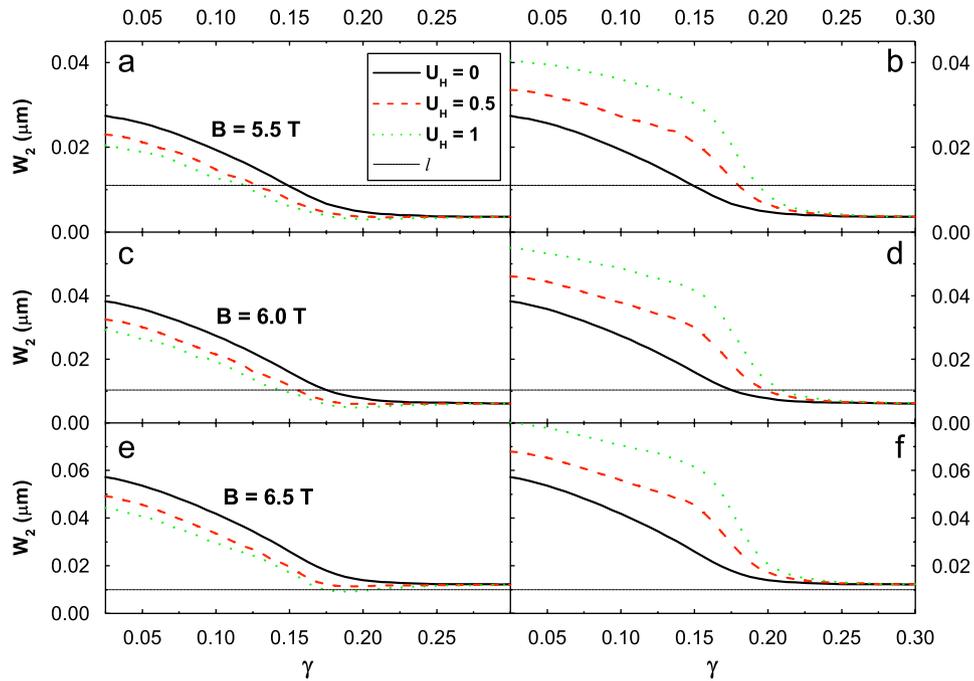
Fig. 2. W_2 at left (left panel) and right (right panel) side of the sample as a function of impurity concentration. At default T and depletion length. Horizontal (dashed–dotted) lines indicate the magnetic length. The sample width is taken to be 3 μm and calculations are done at 3 K.

To summarize, we have studied the widths of the ISs, in the OLR regime. We found that (i) the strong current imposed induces an asymmetry on the IS width depending linearly on the current intensity; (ii) the higher the impurity concentration, the narrower the W_2 is, meanwhile at higher currents this effect becomes more pronounced. The main message of our SC calculations is that the electron velocity strongly depends on the sample parameters and, in addition, in the OLR regime the symmetry between the left and right edges is broken due to electron–electron interaction.

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