Mesoscopic Superposition States in Relativistic Landau Levels

A. Bermudez,[1] M. A. Martin-Delgado,[1] and E. Solano[2,3]

[1]Departamento de Física Teórica I, Universidad Complutense, 28040 Madrid, Spain

[2]Physics Department, ASC, and CeNS, Ludwig-Maximilians-Universität, Theresienstrasse 37, 80333 Munich, Germany

[3]Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado Postal 1761, Lima, Peru

(Received 27 June 2007; published 18 September 2007)

We show that a linear superposition of mesoscopic states in relativistic Landau levels can be built when an external magnetic field couples to a relativistic spin 1/2 charged particle. Under suitable initial conditions, the associated Dirac equation produces unitarily superpositions of coherent states involving the particle orbital quanta in a well-defined mesoscopic regime. We demonstrate that these mesoscopic superpositions have a purely relativistic origin and disappear in the nonrelativistic limit.

DOI: 10.1103/PhysRevLett.99.123602 PACS numbers: 42.50.Pq, 03.65.Pm, 03.67.−a, 42.50.Dv

Linear superpositions of mesoscopic coherent states were introduced to highlight the distinctive fundamental properties of quantum mechanics as opposed to classical theories [1]. However, their reach has gone far beyond and, presently, different subfields of quantum information, like quantum communication, fault-tolerant quantum computation, secret sharing, among others, use them as a central resource [2]. The generation of mesoscopic superpositions based on relativistic quantum effects has not been addressed so far and it is the main purpose of our work. To achieve this goal, we study the Dirac dynamics under specific conditions that we detail below.

A relativistic electron of mass m, charge $-e$, subjected to a constant homogeneous magnetic field along the z axis, is described by means of the Dirac equation

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t} = [c\boldsymbol{\alpha}\cdot(\hat{\mathbf{p}} + e\hat{\mathbf{A}}) + mc^2\beta]|\Psi\rangle, \tag{1}$$

where $|\Psi\rangle$ stands for the Dirac 4-component spinor, $\hat{\mathbf{p}}$ represents the momentum operator, and c the speed of light. Here, $\hat{\mathbf{A}}$ is the vector potential related to the magnetic field through $\hat{\mathbf{B}} = \nabla \wedge \hat{\mathbf{A}}$, and $\beta = \mathrm{diag}(\mathbb{I}, -\mathbb{I})$, $\alpha_j = \text{off-diag}(\sigma_j, \sigma_j)$ are the Dirac matrices in the standard representation with σ_j as the usual Pauli matrices [3]. The energy spectrum of this system is described by the relativistic Landau levels, first derived by Rabi [4]

$$E = \pm\sqrt{m^2c^4 + p_z^2c^2 + 2mc^2\hbar\omega_c(n+1)}, \tag{2}$$

where $n = 0, 1, \ldots$ and $\omega_c = eB/mc$ is the cyclotron frequency describing the electron helicoidal trajectory. Note that relativistic Landau levels have already been considered in the monochromatic wave field [5] and also the excitation of coherent superposition states via multiphoton transitions in the quasimonochromatic wave field [6].

In this Letter, we derive an exact mapping between this relativistic model and a combination of Jaynes-Cummings (JC) and Anti-Jaynes-Cummings (AJC) interactions [7], so widely used by the quantum optics community. This original perspective allows a deeper understanding of relativistic effects [8], as well as the prediction of novel effects such as the existence of relativistic mesoscopic superposition states, constituting the relativistic extension of their nonrelativistic version [1,2].

Working in the axial gauge, where $\hat{\mathbf{A}} := \frac{B}{2}[-\hat{y}, \hat{x}, 0]$, the relativistic Hamiltonian can be expressed as follows

$$\begin{aligned} H_D = {}& mc^2\beta + \alpha_z\hat{p}_z + c\alpha_x(\hat{p}_x - m\omega\hat{y}) \\ & + c\alpha_y(\hat{p}_y + m\omega\hat{x}), \end{aligned} \tag{3}$$

where we have introduced $\omega := \omega_c/2$. It is convenient to introduce the chiral creation-annihilation operators

$$\begin{aligned} \hat{a}_r &:= \frac{1}{\sqrt{2}}(\hat{a}_x - i\hat{a}_y), & \hat{a}_r^\dagger &:= \frac{1}{\sqrt{2}}(\hat{a}_x^\dagger + i\hat{a}_y^\dagger), \\ \hat{a}_l &:= \frac{1}{\sqrt{2}}(\hat{a}_x + i\hat{a}_y), & \hat{a}_l^\dagger &:= \frac{1}{\sqrt{2}}(\hat{a}_x^\dagger - i\hat{a}_y^\dagger), \end{aligned} \tag{4}$$

where $\hat{a}_x^\dagger$, $\hat{a}_x$, $\hat{a}_y^\dagger$, $\hat{a}_y$, are the creation-annihilation operators of the harmonic oscillator $\hat{a}_i^\dagger = \frac{1}{\sqrt{2}}(\frac{1}{\tilde{\Delta}}\hat{r}_i - i\frac{\tilde{\Delta}}{\hbar}\hat{p}_i)$, $i = x, y$, and $\tilde{\Delta} = \sqrt{\hbar/m\omega}$ represents the oscillator's ground state width. Let us first consider an inertial frame $\mathcal{S}'$ which moves along the axis OZ at constant momentum p_z with respect to a rest frame $\mathcal{S}$. In the moving frame, the momentum $p_z' = 0$ in Eq. (3), and using these chiral operators (4), the Dirac Hamiltonian becomes

$$H_D = mc^2\begin{bmatrix} 1 & 0 & 0 & -i2\sqrt{\xi}\hat{a}_r \\ 0 & 1 & i2\sqrt{\xi}\hat{a}_r^\dagger & 0 \\ 0 & -i2\sqrt{\xi}\hat{a}_r & -1 & 0 \\ i2\sqrt{\xi}\hat{a}_r^\dagger & 0 & 0 & -1 \end{bmatrix}, \tag{5}$$

where $\xi := \hbar\omega/mc^2$ is a parameter which controls the nonrelativistic limit. It follows from Eq. (5), that the chiral operator couples different components of the Dirac spinor and simultaneously creates or annihilates right-handed quanta. Expressing the Dirac spinor appropriately $|\Psi\rangle := [\psi_1, \psi_2, \psi_3, \psi_4]^t$, the Hamiltonian becomes

$$H_D = mc^2\sigma_{14}^z + g_{14}\sigma_{14}^+\hat{a}_r + g_{14}^*\sigma_{14}^-\hat{a}_r^\dagger + mc^2\sigma_{23}^z + g_{23}\sigma_{23}^+\hat{a}_r^\dagger + g_{23}^*\sigma_{23}^-\hat{a}_r, \tag{6}$$

where $g_{14} := -i2mc^2\sqrt{\xi} =: -g_{23}$ represent the coupling constants between the different spinor components. The first term in Eq. (6) which couples components $\{\psi_1, \psi_4\}$ is identical to a detuned Jaynes-Cummings interaction

$$H_{\rm JC}^{14} = \Delta\sigma_{14}^z + (g_{14}\sigma_{14}^+\hat{a}_r + g_{14}^*\sigma_{14}^-\hat{a}_r^\dagger). \tag{7}$$

Likewise, the remaining term is identical to a anti-Jaynes-Cummings (AJC) interaction between $\{\psi_2, \psi_3\}$

$$H_{\rm AJC}^{23} = \Delta\sigma_{23}^z + (g_{23}\sigma_{23}^+\hat{a}_r^\dagger + g_{23}^*\sigma_{23}^-\hat{a}_r), \tag{8}$$

with a similar detuning parameter $\Delta := mc^2$. Therefore, the Dirac Hamiltonian is the sum of JC and AJC terms $H_D = H_{\rm JC}^{14} + H_{\rm AJC}^{23}$, which is represented in Fig. 1. This level diagram, so usual in quantum optics, must be interpreted as follows. According to the free Dirac equation $g_{14} = g_{23} = 0$, the spinor components $\{\psi_1, \psi_2\}$ correspond to positive energy components, while $\{\psi_3, \psi_4\}$ stand for negative energy components separated by an energy gap $\Delta\epsilon = 2mc^2$. Furthermore, these components have a well-defined value of the spin projected along the z axis. Namely, $\{\psi_1, \psi_3\}$ are spin-up components while $\{\psi_2, \psi_4\}$ represent spin-down components. Thus, as Fig. 1 states, the interaction of a free electron with a constant magnetic field induces transitions between spin-up (spin-down) and positive (negative) energy components. Each transition between the large and small components $\{\psi_1, \psi_2\} \leftrightarrow \{\psi_3, \psi_4\}$ is accompanied by a spin flip and mediated through the creation or annihilation of right-handed quanta of rotation. The whole Hilbert space can be divided into a set of invariant subspaces, which facilitate the diagonalization task. In order to do so, let us introduce the states $|j, n_r\rangle = |j\rangle|n_r\rangle$, which represent the electronic spinor component ψ_j and the electronic rotational state $|n_r\rangle := \frac{1}{\sqrt{n_r!}}(\hat{a}_r^\dagger)^{n_r}|\text{vac}\rangle$. Because of the previously described mapping (6), the Hilbert space can be described as $\mathcal{H} = \tilde{\mathcal{H}} \bigoplus_{n_r=0}^{\infty} \mathcal{H}_{n_r}$, where $\tilde{\mathcal{H}}$ is spanned by states $\tilde{\mathcal{H}} = \text{span}\{|4, 0\rangle, |2, 0\rangle\}$, which have energies $\tilde{E} := \pm\Delta = \pm mc^2$, respectively. These states can be interpreted as quantum optical dark states, since they do not evolve

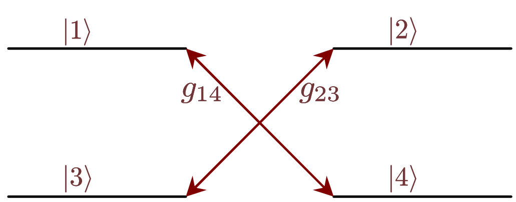

FIG. 1 (color online). Quantum optical representation of the relativistic e^- levels coupled by means of a constant magnetic field.

exchanging chiral quanta (6). The remaining invariant subspaces are

$$\mathcal{H}_{n_r} = \text{span}\{|1, n_r\rangle, |4, n_r + 1\rangle, |2, n_r + 1\rangle, |3, n_r\rangle\}. \tag{9}$$

and allow a block decomposition of the Hamiltonian (5)

$$H_{n_r} = \begin{bmatrix} \Delta & -g\sqrt{n_r+1} & 0 & 0 \\ -g^*\sqrt{n_r+1} & -\Delta & 0 & 0 \\ 0 & 0 & \Delta & g\sqrt{n_r+1} \\ 0 & 0 & g^*\sqrt{n_r+1} & -\Delta \end{bmatrix}, \tag{10}$$

where $g = i2mc^2\sqrt{\xi}$ is related to the coupling constants introduced in Eq. (6). This Hamiltonian can be block diagonalized, yielding the following energies

$$E' = \pm E'_{n_r} := \pm\sqrt{\Delta^2 + |g|^2(n_r + 1)}, \tag{11}$$

which correspond to the relativistic Landau levels in Eq. (2) with $p_z = 0$. In the nonrelativistic limit, where $E'_{n_r} = mc^2 + \epsilon'_{n_r}$ such that $\epsilon'_{n_r} \ll mc^2$, we find that the energy spectrum in Eq. (11) can be expressed as $\epsilon'_{n_r} \approx \hbar\omega_c(n_r + 1)$, which are the usual Landau levels [9]. The associated relativistic eigenstates are

$$\begin{aligned} |\pm E'_{n_r}, 1\rangle &:= c_{n_r}^{\pm}|n_r\rangle\chi_{1\uparrow} \mp ic_{n_r}^{\mp}|n_r + 1\rangle\chi_{2\downarrow}, \\ |\pm E'_{n_r}, 2\rangle &:= c_{n_r}^{\pm}|n_r + 1\rangle\chi_{1\downarrow} \mp ic_{n_r}^{\mp}|n_r\rangle\chi_{2\uparrow}, \end{aligned} \tag{12}$$

where we have introduced the usual Pauli spinors $\chi_{1\uparrow} := (1, 0, 0, 0)^t$, $\chi_{1\downarrow} := (0, 1, 0, 0)^t$, $\chi_{2\uparrow} := (0, 0, 1, 0)^t$, $\chi_{2\downarrow} := (0, 0, 0, 1)^t$, and $c_{n_r}^{\pm} := \sqrt{(E'_{n_r} \pm mc^2)/2E'_{n_r}}$. The rotational and spinorial properties of the eigenstates in Eq. (12) become unavoidably entangled in the moving inertial frame $\mathcal{S}'$. To obtain the corresponding solutions in the rest frame $\mathcal{S}$, we must perform a Lorentz boost along the OZ axis $p'^\mu := [E'/c, p'^x, p'^y, 0] \rightarrow p^\mu := [E/c, p^x, p^y, p^z]$. Considering the invariance of the four-momentum $g_{\mu\nu}p^\mu p^\nu = g_{\mu\nu}p'^\mu p'^\nu$, where the Minkowski metric tensor is $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and that $p^x = p'^x$,$p^y = p'^y$, we come to $E'^2/c^2 = E^2/c^2 - p_z^2$. Substituting in Eq. (11)

$$E = \pm E_{n_r} := \pm\sqrt{\Delta^2 + p_z^2c^2 + |g|^2(n_r + 1)}. \tag{13}$$

These are the relativistic Landau levels in Eq. (2), whose associated eigenstates may be obtained by means of a Lorentz boost to the Dirac spinor $\Psi(x^\mu) = S_{L_3}^{-1}\Psi'(x'^\mu)$

$$S_{L_3}^{-1} = \cosh\frac{\eta}{2}\begin{bmatrix} 1 & 0 & \tanh\frac{\eta}{2} & 0 \\ 0 & 1 & 0 & -\tanh\frac{\eta}{2} \\ \tanh\frac{\eta}{2} & 0 & 1 & 0 \\ 0 & -\tanh\frac{\eta}{2} & 0 & 1 \end{bmatrix}, \tag{14}$$

where η is the rapidity, $\cosh\eta/2 = \sqrt{(E_{n_r} + E'_{n_r})/2E'_{n_r}}$, $\tanh\eta/2 = p_zc/(E_{n_r} - E'_{n_r})$. With these expressions, one

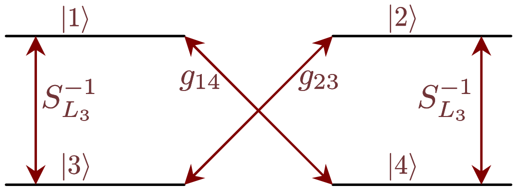

FIG. 2 (color online). Quantum optical representation of the coupling between the relativistic levels caused by the Lorentz boost.

can finally obtain the eigenstates in the rest frame $\mathcal{S}$

$$\begin{aligned} |\pm E_{n_r}, 1\rangle &:= c_{n_r}^{\pm}|n_r\rangle\left(\cosh\frac{\eta}{2}\chi_{1\uparrow} + \sinh\frac{\eta}{2}\chi_{2\uparrow}\right) \\ &\quad + \pm i c_{n_r}^{\mp}|n_r+1\rangle\left(\sinh\frac{\eta}{2}\chi_{1\downarrow} - \cosh\frac{\eta}{2}\chi_{2\downarrow}\right), \\ |\pm E_{n_r}, 2\rangle &:= c_{n_r}^{\pm}|n_r+1\rangle\left(\cosh\frac{\eta}{2}\chi_{1\downarrow} - \sinh\frac{\eta}{2}\chi_{2\downarrow}\right) \\ &\quad + \mp i c_{n_r}^{\mp}|n_r\rangle\left(\sinh\frac{\eta}{2}\chi_{1\uparrow} + \cosh\frac{\eta}{2}\chi_{2\uparrow}\right), \end{aligned} \tag{15}$$

where the four spinor components get mixed in the rest frame $\mathcal{S}$ due to the Lorentz boost (see Fig. 2).

Once the relativistic eigenstates have been obtained in a quantum optics framework, we can discuss a novel aspect of the relativistic electron dynamics, the rise of relativistic mesoscopic superposition states. Our goal now is to find the conditions guaranteeing the existence of such states, which follow from the quantum optical (6) mapping and will turn out to be nontrivial.

For the sake of simplicity we restrict to the regime with $p_z = 0$, where the effective dynamics of an initial state $|\Psi(0)\rangle = |z_r\rangle\chi_{1\uparrow}$, with $|z_r\rangle := e^{-(1/2)|z_r|^2}\sum_{n_r=0}^{\infty}\frac{z_r^{n_r}}{\sqrt{n_r!}}|n_r\rangle$ being a right-handed coherent state with $z_r \in \mathbb{C}$, can be described solely by the JC term (7). Due to the invariance of Hilbert subspaces, a blockade of the AJC term occurs (see Fig. 3), and three different regimes appear:

Macroscopic Regime.—In this regime, the mean number of right-handed quanta $\bar{n}_r = |z_r|^2 \to \infty$, so the discreteness of the orbital degree of freedom can be neglected. Setting $z_r = i|z_r|$, the JC term (7) can be approximately described by the semiclassical Hamiltonian

$$H_{14}^{\rm sc} = \Delta\sigma_z + |g||z_r|(\sigma^+ + \sigma^-), \tag{16}$$

whose energies are $E^{\rm sc} = \pm E_{z_r} := \pm\sqrt{\Delta^2 + |g|^2|z_r|^2}$. This semiclassical energy levels resemble the original spectrum (11), but the corresponding

$$|\pm E_{z_r}\rangle := c_{z_r}^{\pm}\chi_{1\uparrow} \pm i c_{z_r}^{\mp}\chi_{2\downarrow}, \tag{17}$$

with $c_{z_r}^{\pm} := \sqrt{(E_{z_r} \pm \Delta)/2E_{z_r}}$, are clearly different from those in Eq. (12). In the semiclassical limit, entanglement between the spin and orbital degrees of freedom is absent. The state $|\Psi(0)\rangle := \chi_{1\uparrow}$ evolves according to

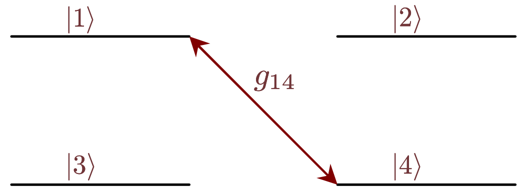

FIG. 3 (color online). Blockade of the AJC coupling.

$$\begin{aligned} |\Psi(t)\rangle &= \left(\cos\Omega_{z_r}^{\rm sc}t - \frac{i}{\sqrt{1+4\xi\bar{n}_r}}\sin\Omega_{z_r}^{\rm sc}t\right)\chi_{1\uparrow} \\ &\quad + +i\left(\sqrt{\frac{4\xi\bar{n}_r}{1+4\xi\bar{n}_r}}\sin\Omega_{z_r}^{\rm sc}t\right)\chi_{2\downarrow}, \end{aligned} \tag{18}$$

where $\Omega_{z_r}^{\rm sc} := E_{z_r}/\hbar$ is the semiclassical Rabi frequency. Therefore, relativistic mesoscopic superpositions cannot be produced in this manner.

Microscopic regime.—In this limit, $\bar{n}_r = |z_r|^2 \lesssim 10$ is small enough for the discreteness of the orbital degree of freedom to become noticeable. Especially interesting is the evolution of the vacuum of right-handed quanta

$$\begin{aligned} |\Psi(t)\rangle &= \left(\cos\omega_0 t - \frac{i}{\sqrt{1+4\xi}}\sin\omega_0 t\right)|0\rangle\chi_{1\uparrow} \\ &\quad + +\left(\sqrt{\frac{4\xi}{1+4\xi}}\sin\omega_0 t\right)|1\rangle\chi_{2\downarrow}, \end{aligned} \tag{19}$$

where $\omega_0 := \frac{mc^2}{\hbar}\sqrt{1+4\xi}$ is the vacuum Rabi frequency. We observe how the spinorial and orbital degrees of freedom become inevitably entangled as time evolves due to the interference of positive and negative energy solutions, i.e., *zitterbewegung* [10]. This behavior is crucial for the generation of mesoscopic superpositions, although their growth cannot occur under this regime since the orbital degree of freedom are not of a mesoscopic nature.

Mesoscopic regime.—When the mean number of orbital quanta $10 \lesssim \bar{n}_r \lesssim 100$ attains a mesoscopic value, collapses and revivals in the Rabi oscillations (19) occur [11]. An asymptotic approximation accounting for the collapse-revival phenomenon has been derived [12,13], and its validity has been experimentally tested in cavity QED [14]. Below, we derive a relativistic mesoscopic approximation, which allows us to predict the generation of relativistic mesoscopic superposition states.

Let us first discuss this asymptotic approximation, where the semiclassical eigenstates (17) play an essential role. The states $|\Psi^{\pm}(0)\rangle := |\pm E_{z_r}\rangle|z_r\rangle$ evolve as

$$|\Psi^{\pm}(t)\rangle \approx \left(c_{z_r}^{\pm}e^{\mp i[|g|^2/(2\hbar E_{z_r})]t}\chi_{1\uparrow} \pm i c_{z_r}^{\mp}\chi_{2\downarrow}\right)e^{\mp i\hat{\Theta}t}|z_r\rangle, \tag{20}$$

where $\hat{\Theta} := \frac{1}{\hbar}\sqrt{\Delta^2 + |g|^2\hat{a}_r^\dagger\hat{a}_r}$ depends on the chiral operators. The electron spin and orbital degrees of freedom remain disentangled throughout the whole evolution

$|\Psi^{\pm}(t)\rangle = |\Phi_{\rm sp}^{\pm}(t)\rangle \otimes |\Phi_{\rm orb}^{\pm}(t)\rangle$. This peculiar behavior may be compared to the *zitterbewegung* oscillations in Eq. (19), where entanglement plays a major role.

For times shorter than the usual revival time $t \ll t_R := 2\pi E_{z_r}\hbar/|g|^2$, the asymptotic approximation in Eq. (20) can be pushed further, and a suggestive expression for the evolved orbital state $|\Phi_{\rm orb}^{\pm}(t)\rangle := e^{\mp i\Theta t}|z_r\rangle$ follows

$$|\Phi_{\rm orb}^{\pm}(t)\rangle \approx e^{\mp i(t/\hbar)(E_{z_r}-(|g|^2|z_r|^2/2E_{z_r}))}|z_r e^{\mp i(|g|^2 t/2\hbar E_{z_r})}\rangle. \tag{21}$$

Up to an irrelevant global phase, the short time evolution of the orbital coherent state yields another coherent state whose phase evolves in time according to Eqs. (21). Considering the position operators $\hat{X} = \tilde{\Delta}(\hat{a}_r + \hat{a}_r^{\dagger} + \hat{a}_l + \hat{a}_l^{\dagger})/2$, $\hat{Y} = i\tilde{\Delta}(\hat{a}_r - \hat{a}_r^{\dagger} - \hat{a}_l + \hat{a}_l^{\dagger})/2$, we calculate the expectation value that describes the electron trajectory $\langle\hat{\mathbf{X}}(t)\rangle_{\pm} := (\langle\hat{X}(t)\rangle_{\pm}, \langle\hat{Y}(t)\rangle_{\pm})$, yielding the following:

$$\langle\hat{\mathbf{X}}(t)\rangle_{\pm} = \tilde{\Delta}|z_r|(\mp \sin\Omega_{\rm rot}t, + \cos\Omega_{\rm rot}t), \tag{22}$$

where $\Omega_{\rm rot} := |g|^2/2E_{z_r}\hbar$. Therefore solutions $|\Psi^{+}\rangle$ rotate counterclockwise around the z axis, while $|\Psi^{-}\rangle$ rotate clockwise. Considering $|\Psi(0)\rangle := \chi_{1,\uparrow}|z_r\rangle = (c_{z_r}^{+}|+E_{z_r}\rangle + c_{z_r}^{-}|-E_{z_r}\rangle)|z_r\rangle$, which involves both semiclassical solutions (17), it splits up in two components which rotate in opposite directions as time elapses

$$|\Psi(t)\rangle = c_{z_r}^{+}|\Phi_{\rm sp}^{+}(t)\rangle|\Phi_{\rm orb}^{+}(t)\rangle + c_{z_r}^{-}|\Phi_{\rm sp}^{-}(t)\rangle|\Phi_{\rm orb}^{-}(t)\rangle, \tag{23}$$

where we have introduced the spinor states for clarity $|\Phi_{\rm sp}^{\pm}(t)\rangle := (c_{z_r}^{+}e^{\mp i(|g|^2/(2\hbar E_{z_r}))t}\chi_{1\uparrow} \pm ic_{z_r}^{-}\chi_{2\downarrow})$. Once we have discussed the relativistic asymptotic approximation (23), we can proceed with the generation of relativistic mesoscopic superpositions. In order to produce them, we need

$$|\Phi_{\rm sp}^{+}(t_d)\rangle = e^{i\delta}|\Phi_{\rm sp}^{-}(t_d)\rangle =: |\tilde{\Phi}_d\rangle \tag{24}$$

to be fulfilled, where t_d is the required interaction time and $\delta \in \mathbb{R}$. If such a constraint (24) is satisfied, then the time evolution (23) under the mesoscopic regime leads to

$$|\Psi(t_d)\rangle = |\tilde{\Phi}_d\rangle(c_{z_r}^{+}|\Phi_{\rm orb}^{+}(t)\rangle + e^{i\delta}c_{z_r}^{-}|\Phi_{\rm orb}^{-}(t)\rangle), \tag{25}$$

and we obtain a coherent superposition of states in the orbital degree of freedom. Furthermore, using the properties of unitary evolution, it follows that

$$\langle +E_{z_r}| - E_{z_r}\rangle = 0 \mapsto \langle\Phi_{\rm orb}^{+}(t_d)|\Phi_{\rm orb}^{-}(t_d)\rangle = 0, \tag{26}$$

and therefore the orbital state in Eq. (25),

$$|\Phi_{\rm orb}^{\rm ms}\rangle := c_{z_r}^{+}|\Phi_{\rm orb}^{+}(t_d)\rangle + e^{i\delta}c_{z_r}^{-}|\Phi_{\rm orb}^{-}(t_d)\rangle, \tag{27}$$

represents a superposition of mesoscopically distinct states in the relativistic scenario. We can verify the correct generation of these states with condition (24). At half revival time $t_d = t_R/2 = \pi E_{z_r}\hbar/|g|^2$, we find

$$|\langle\Phi_{\rm sp}^{+}(t_d)|\Phi_{\rm sp}^{-}(t_d)\rangle| \approx \sqrt{\frac{4\xi\bar{n}_r}{1 + 4\xi\bar{n}_r}}. \tag{28}$$

In order to satisfy the aforementioned constraint, one must take the ultrarelativistic limit $\xi \gg 1/\bar{n}_r$, where $|\langle\Phi_{\rm sp}^{+}(t_d)|\Phi_{\rm sp}^{-}(t_d)\rangle| \approx 1 + \mathcal{O}(\frac{1}{\bar{n}_r})$, and thus a relativistic mesoscopic superposition of coherent states is produced. In the nonrelativistic scenario Eq. (28) yields $|\langle\Phi_{\rm sp}^{+}(t_s)|\Phi_{\rm sp}^{-}(t_s)\rangle| \approx 2\sqrt{\xi\bar{n}_r} + \mathcal{O}(\xi^{3/2}) \ll 1$, and the generation condition cannot be fulfilled in this case. As the electron slows down, the coherence of Eq. (25) vanishes, as well as the relativistic mesoscopic superposition.

In conclusion, we have predicted the generation of mesoscopic superposition states in the relativistic Landau levels via a suitable unitary Dirac dynamics.

We thank financial support from FPU M.E.C. grant (A. B.), EU Project INSTANS (M. A. MD.), DGS under Contract BFM No. 2003-05316-C02-01 and CAM-UCM under Ref. 910758 (A. B., M. A. MD.), and from DFG No. SFB 631, EU EuroSQIP projects, and the German Excellence Initiative via the "Nanosystems Initiative Munich" (E. S.).

[1] E. Schrödinger, Naturwissenschaften **23**, 807812 (1935); **23**, 823828 (1935); **23**, 844849 (1935).
[2] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[3] W. Greiner, *Relativistic Quantum Mechanics. Wave Equations* (Springer, Berlin, 2000).
[4] I. I. Rabi, Z. Phys. **49**, 507 (1928).
[5] P. J. Redmond, Math. Phys. Appl. Math. **6**, 1163 (1965); I. M. Ternov, V. R. Khalilov, and V. N. Rodionov, *Interaction of Charged Particles with Strong Electromagnetic Field* (Mosk. Gos. Univ., Moscow, 1982) (in Russian).
[6] H. K. Avetissian, *Relativistic Nonlinear Electrodynamics* (Springer, New York, 2006).
[7] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).
[8] M. H. Johnson and B. A. Lippmann, Phys. Rev. **76**, 828 (1949); **77**, 702 (1950).
[9] L. Landau, Z. Phys. **64**, 629 (1930).
[10] A. Bermudez, M. A. Martin-Delgado, and E. Solano, arXiv:0704.2315.
[11] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980).
[12] J. Gea-Banacloche, Phys. Rev. A, **44**, 5913 (1991).
[13] J. Gea-Banacloche, Opt. Commun. **88**, 531 (1992).
[14] A. Auffeves *et al.*, Phys. Rev. Lett. **91**, 230405 (2003).