SU(4) Fermi Liquid State and Spin Filtering in a Double Quantum Dot System

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We study a symmetrical double quantum dot (DD) system with strong capacitive interdot coupling using renormalization group methods. The dots are attached to separate leads, and there can be a weak tunneling between them. In the regime where there is a single electron on the DD the low-energy behavior is characterized by an SU(4)-symmetric Fermi liquid theory with entangled spin and charge Kondo correlations and a phase shift $\pi/4$. Application of an external magnetic field gives rise to a large magnetoconductance and a crossover to a purely charge Kondo state in the charge sector with SU(2) symmetry. In a four-lead setup we find perfectly spin-polarized transmission.

Introduction.—Quantum dots are one of the most basic building blocks of mesoscopic circuits [1]. In many respects quantum dots act as large complex atoms coupled to conducting leads that are used to study transport. The physical properties of these dots depend essentially on the level spacing and precise form of the coupling to the leads: They can exhibit Coulomb blockade phenomena [2], build up correlated Kondolike states of various kinds [3–5], or develop conductance fluctuations.

The simplest mesoscopic circuits that go beyond single dot devices in their complexity are double dot (DD) devices (see Fig. 1). These "artificial molecules" have been extensively studied both theoretically [6–11] and experimentally [12–15]: They may give rise to stochastic Coulomb blockade [6] and peak splitting [7,12], can be used as single electron pumps [1], were proposed to measure high frequency quantum noise [11], and are building blocks for more complicated mesoscopic devices such as turnstiles or cellular automata [16]. DDs also have interesting degeneracy points where quantum fluctuations may lead to unusual strongly correlated states [17].

In the present Letter we focus our attention to small semiconducting DDs with large interdot capacitance [10,17]. We consider the regime where the gate voltages $V_\pm$ are such that the lowest lying charging states, $(n_+, n_-) = (0, 1)$ and $(1, 0)$, are almost degenerate: $E(1, 0) - E(0, 1) \approx 0$ [$n_\pm =$ No. of extra electrons on dot "$\pm$", and $E(n_+, n_-)$ is measured from the common chemical potential of the two leads]. We consider the simplest, most common case where the states $(1, 0)$ and $(0, 1)$ have both spin $S = 1/2$, associated with the extra electron on the dots. Then at energies below the charging energy of the DD, $\tilde{E}_C \equiv \min\{E(1, 1) - E(0, 1), E(0, 0) - E(0, 1)\}$, the dynamics of the DD is restricted to the subspace $\{S^z = \pm 1/2; n_+ - n_- = \pm 1\}$.

As we discuss below, quantum fluctuations between these four quantum states of the DD generate an unusual strongly correlated Fermi liquid state, where the spin and charge degrees of freedom of the DD are totally entangled. We show that this state possesses an SU(4) symmetry corresponding to the total internal degrees of freedom of the DD, and is characterized by a phase shift $\delta = \pi/4$. This phase shift can be measured by integrating the DD device in an Aharonov-Bohm interferometer [18]. Application of an external field on the DD suppresses spin fluctuations. However, charge fluctuations are unaffected by the magnetic field and still give rise to a Kondo effect in the charge (orbital) sector [10,17,19]. We show that in a four-lead setup this latter state gives rise to an almost *totally spin-polarized* current through the DD with a field-independent conductance $G \approx e^2/h$. The conductance *across* the dots, on the other hand, shows a large *negative magnetoresistance* at $T = 0$ temperature.

Model.—We first discuss the setup in Fig. 1. At energies below $\tilde{E}_C$ we describe the isolated DD in terms of the
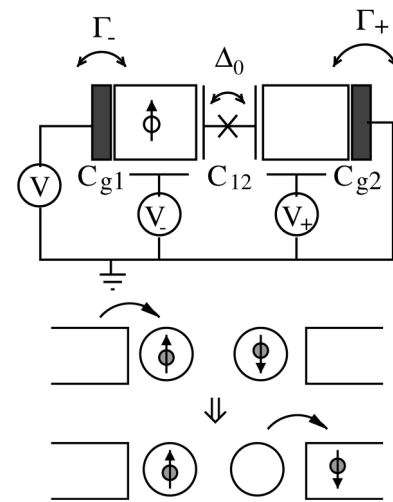

FIG. 1. Top: Schematics of the DD device. Bottom: Virtual process leading to "spin-flip assisted tunneling" as described in Eq. (4).

orbital pseudospin $T^z \equiv (n_+ - n_-)/2 = \pm\frac{1}{2}$:

$$H_{\text{dot}} = -\delta E\, T^z - t\, T^x - B\, S^z. \tag{1}$$

The term proportional to T^z describes the *energy difference* of the two charge states [$\delta E \equiv E(1,0) - E(0,1) \sim V_+ - V_-$ for a fully symmetrical system], while $t \ll \tilde{E}_C$ is the tunneling amplitude between them. The last term stands for the Zeeman splitting due to an applied local magnetic field in the z direction. We are interested in the regime, where—despite the large capacitive coupling—the tunneling between the dots is small. Furthermore, one needs a large enough single particle level spacing Δ on the dots. Both conditions can be satisfied by making small dots [20], which are close together or capacitively coupled to a common top-gate electrode [21].

The leads are described by the Hamiltonian:

$$H_{\text{leads}} = \sum_{|\varepsilon|<D} \epsilon\, a^\dagger_{\varepsilon\sigma+} a_{\varepsilon\sigma+} + \sum_{|\varepsilon|<D} \epsilon\, a^\dagger_{\varepsilon\sigma-} a_{\varepsilon\sigma-}, \tag{2}$$

where $a^\dagger_{\varepsilon\sigma+}$ ($a^\dagger_{\varepsilon\sigma-}$) creates an electron in the right (left) lead with energy ε and spin σ, $D \sim \min\{\tilde{E}_C, \Delta\} \equiv 1$ is a cutoff, and $\{a^\dagger_{\varepsilon\sigma\tau}, a_{\varepsilon'\sigma'\tau'}\} = \delta_{\tau\tau'}\delta_{\sigma\sigma'}\delta(\varepsilon - \varepsilon')$.

To determine the effective DD-lead coupling we have to consider virtual charge fluctuations to the excited states with $n_+ + n_- = 0$ and 2, generated by tunneling from the leads to the dots. By second order perturbation theory in the lead-dot tunneling we obtain the following effective Hamiltonian:

$$H_{\text{Kondo}} = \frac{1}{2}J_+ P_+ \vec{S}(\psi^\dagger\, \vec{\sigma}\, p_+ \psi) + \frac{1}{2}J_- P_- \vec{S}(\psi^\dagger\, \vec{\sigma}\, p_- \psi), \tag{3}$$

$$H_{\text{assist}} = Q_\perp [T^+ \vec{S}(\psi^\dagger\vec{\sigma}\tau^-\psi) + \text{h.c.}], \tag{4}$$

$$H_{\text{orb}} = \frac{1}{2}\{V_z T^z(\psi^\dagger\tau^z\psi) + V_\perp[T^+(\psi^\dagger\tau^-\psi) + \text{h.c.}]\}, \tag{5}$$

where $\psi_{\sigma\tau} = \int d\varepsilon\, a_{\varepsilon\sigma\tau}$ and $\vec{\sigma}$ and $\vec{\tau}$ denote the spin and *orbital pseudospin* of the electrons ($\sigma = \uparrow, \downarrow$; $\tau = \tau^z = \pm 1$). The operators $P_\pm = (1 \pm 2T^z)/2$ and $p_\pm = (1 \pm \tau^z)/2$ project out the DD states $(1,0)$ and $(0,1)$, and the right/left lead channels, respectively.

In the limit of small dot-lead tunneling the dimensionless exchange couplings are $J_\pm \sim \Gamma_\pm/\tilde{E}_C$ with $\Gamma_\pm$ the tunneling rate to the right (left) lead [22]. The "spin-flip assisted tunneling" $Q_\perp \sim \sqrt{\Gamma_+\Gamma_-}/\tilde{E}_C$ in Eq. (4) gives simultaneous spin- and pseudospin-flip scattering and is produced by virtual processes depicted in the lower part of Fig. 1, while the spin-independent parts of such virtual processes lead to the orbital Kondo term in Eq. (5) with similar amplitudes.

We first focus on the case of a fully symmetrical DD. Then the sum of Eqs. (3) and (4) can be rewritten as

$$\begin{aligned} H_{\text{Kondo}} + H_{\text{assist}} = \frac{1}{2}J\vec{S}(\psi^\dagger\vec{\sigma}\psi) + Q_z T^z\vec{S}(\psi^\dagger\tau^z\vec{\sigma}\psi) \\ + Q_\perp[T^+\vec{S}(\psi^\dagger\tau^-\vec{\sigma}\psi) + \text{h.c.}], \end{aligned} \tag{6}$$

where $J = Q_z = (J_+ + J_-)/4$. The couplings in Eqs. (3)–(5) are not entirely independent, but are related by the constraints $V_\perp = Q_\perp$ and $J = Q_z$.

Scaling Analysis.—The perturbative scaling analysis follows that of a related model in Ref. [23]. In the perturbative RG one performs the scaling by integrating out conduction electrons with energy larger than a scale $\tilde{D} \ll D$, and thus obtains an effective Hamiltonian that describes the physics at energies $\tilde{D}$. For zero δE, t, and B, in the leading logarithmic approximation we find that all couplings diverge at the Kondo temperature $T_K^{(0)}$, where the perturbative scaling breaks down. Nevertheless, the *structure* of the divergent couplings suggests that at low energies $J = V_\perp = V_z = Q_\perp = Q_z$. Thus at small energies—apart from a trivial potential scattering—the effective model is a remarkably simple SU(4) symmetrical exchange model:

$$H_{\text{eff}}(T \to 0) = \tilde{J} \sum_{\alpha,\beta=1,\ldots,4} \psi^\dagger_\alpha \psi_\beta |\beta\rangle\langle\alpha|, \tag{7}$$

where α labels the four combinations of spin and pseudospin indices, and the $|\alpha\rangle$'s denote the DD states. This can be more rigorously proven too using strong coupling expansion, conformal field theory, and large f (flavor) expansion techniques [24–26], and is also confirmed by our numerical computations.

Numerical Renormalization Group (NRG).—To access the low-energy physics of the DD, we used Wilson's NRG approach [27]. In this method one defines a series of rescaled Hamiltonians, H_N, related by the relation [27]:

$$H_{N+1} \equiv \Lambda^{1/2}H_N + \sum_{\sigma\tau} \xi_N(f^\dagger_{N,\sigma\tau}f_{N+1,\sigma\tau} + \text{h.c.}), \tag{8}$$

where $f_{0\sigma\tau} = \psi_{\sigma\tau}/\sqrt{2}$ and $H_0 \equiv 2\Lambda^{1/2}/(1 + \Lambda)H_{\text{int}}$ with $\Lambda \sim 3$ as discretization parameter, and $\xi_N \approx 1$. (For the definition of f_N see Ref. [27].) We have defined $H_{\text{int}} = H_{\text{dot}} + H_{\text{Kondo}} + H_{\text{assist}} + H_{\text{orb}}$. The original Hamiltonian is related to the H_N's as $H = \lim_{N\to\infty} \omega_N H_N$ with $\omega_N = \Lambda^{-(N+1)/2}(1 + \Lambda)/2$. Solving Eq. (8) iteratively we can then use the eigenstates of H_N to calculate physical quantities at a scale $T, \omega \sim \omega_N$.

Results.—First let us consider the case $H_{\text{dot}} = 0$.

Fixed point structure.—The finite size spectrum produced by the NRG procedure contains a lot of information. Among others, we can identify the structure of the low-energy effective Hamiltonian from it [27], and also determine all scattering phase shifts.

In particular, we find that for $\delta E = t = B = 0$ the entire finite size spectrum can be understood as a sum of four independent, spinless chiral fermion spectra with phase shifts $\delta = \pi/4$. This phase shift is characteristic for the SU(4) Hamiltonian, Eq. (7), and simply follows from the Friedel sum rule [24]. Application of an external magnetic field B to the DD gradually shifts δ to the values $\delta_\uparrow \to \pi/2$ and $\delta_\downarrow \to 0$ [28].

Spectral functions.—To learn more about the dynamics of the DD we computed at $T = 0$ the spin spectral

function $\varrho_S^z(\omega) = -(1/\pi)\mathrm{Im}\{\chi_S^z(\omega)\}$, and pseudospin spectral function $\varrho_\tau^y(\omega) = -(1/\pi)\mathrm{Im}\{\chi_\tau^y(\omega)\}$ by the density matrix NRG method [29].

At $B = 0$ the various spectral functions exhibit a peak at the same energy, $T_K^{(0)}$, corresponding to the formation of the SU(4) symmetric state (see Fig. 2). Below $T_K^{(0)}$ all spectral functions become linear, characteristic to a Fermi liquid state with local spin and pseudospin susceptibilities $\chi_S \sim \chi_T \sim 1/T_K^{(0)}$, where the SU(4) "hyperspin" of the dot electron (formed by $\{\uparrow+, \downarrow+, \uparrow-, \downarrow-\}$ components) is screened by the lead electrons.

Now let us consider the case $H_{\mathrm{dot}} \neq 0$. In a large magnetic field, $T_K^{(0)} \ll B$, spin-flip processes are suppressed: The spin spectral function therefore shows only a Schottky anomaly at $\omega \sim B$. Nevertheless, the couplings $V_\perp$ and V_z still generate a purely orbital Kondo state in the spin channel with the same orientation as the DD spin, with a reduced Kondo temperature $T_K(B) < T_K^{(0)}$, and a corresponding phase shift $\delta_\uparrow = \pi/2$.

Because of the spin-pseudospin symmetric structure of the Hamiltonian, Eq. (6), the opposite effect occurs for a large δE: In that limit the charge is localized on one side of the DD, charge fluctuations are suppressed, and the system scales to a spin Kondo problem. A large tunneling, $t > T_K^{(0)}$ is also expected to lead to a somewhat similar effect, though the conductance through the DD behaves very differently in the two cases [28].

dc Conductivity.—First we focus on the conductivity *across the DD* assuming a small tunneling t. Then we can assume that the two dots are in equilibrium with the leads connected to them, and we can compute the induced current perturbatively in t. A simple calculation yields the following formula [30]:
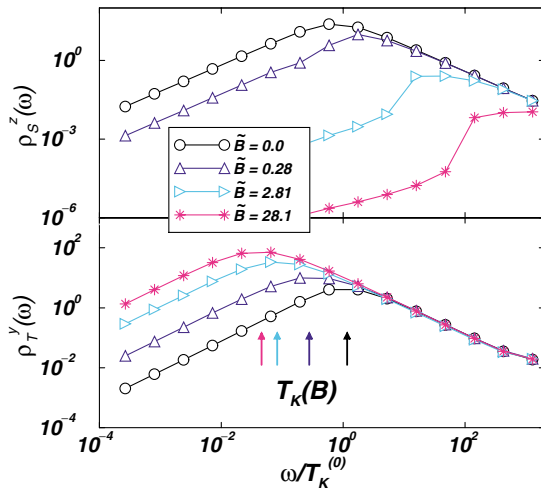

FIG. 2 (color online). $T = 0$ spin and pseudospin spectral functions for $J = Q_z = V_z = 0.14$, $V_\perp = Q_\perp = 0.13$, and various values of $\tilde{B} \equiv B/T_K^{(0)}$. For $B = 0$ both spectral functions exhibit $\sim \omega$ behavior below the Kondo temperature $T_K^{(0)} \sim 10^{-3}$. Applying a magnetic field the situation changes: The $B > T_K^{(0)}$ magnetic field destroys the spin Kondo correlations and leads to a purely orbital Kondo effect.

$$G = \frac{2\pi^2 e^2}{h} t^2 \lim_{\omega \to 0} \frac{\varrho_\tau^y(\omega)}{\omega}. \qquad (9)$$

The normalized dc conductance at $T = 0$ temperature is shown in Fig. 3. Below the orbital Kondo temperature $\varrho_\tau^y(\omega) \sim \omega/T_K^2(B)$, leading to a dimensionless conductance $\sim [t/T_K(B)]^2$. However, $T_K(B)$ strongly decreases with increasing B implying a *large negative magnetoresistance* in the $T = 0$ dc conductance. This effect is related to the correlation between spin and orbital degrees of freedom. We have to emphasize that the simple considerations above only apply in the regime $t \ll T_K(B)$. For larger values of t a more complete calculation is needed.

Having extracted the phase shifts from the NRG spectra, we can construct the scattering matrix in more general geometries too and compute the $T = 0$ conductance using the Landauer-Buttiker formula [28,31,32]. In the perfectly symmetrical two terminal four-lead setup of Fig. 4 with $\delta E = t = 0$, e.g., the dc conductance is $G_{13} = \frac{1}{2} G_Q\{\sin^2[\delta_\downarrow(B)] + \sin^2[\delta_\uparrow(B)]\}$, where $G_Q = 2e^2/h$ is the quantum conductance. By the Friedel sum rule $\delta_\uparrow(B) = \pi/2 - \delta_\downarrow(B)$, and thus $G_{13}(T = 0) = G_Q/2$, independently of B. However, the polarization of the transmitted current, $P = 2\sin^2(\delta_\uparrow) - 1$ tends rapidly to one as $B > T_K^{(0)}$, and the DD thereby acts as a *perfect spin filter* at $T = 0$ with $B > T_K^{(0)}$, and could also serve as a spin pump. For a typical $T_K \approx 0.5$ K and a g factor $g \approx -0.4$ as in GaAs, e.g., a field of $2.5T$ would give a 97% polarized current, comparable to other spin filter designs [33]. Lowering T_K even higher polarizations could be obtained.

Robustness.—Since the spin S^α and pseudospin T^α are both marginal operators at the SU(4) fixed point [25], we conclude that the SU(4) behavior is stable in the sense that a small but finite value of $\delta E, B, t \ll T_K^{(0)}$ will lead only to small changes in physical properties such as the phase shifts. The anisotropy of the couplings is also irrelevant in the RG sense [25,26], and the role of $J_- \neq J_+$ symmetry breaking is only to renormalize the bare value of δE, which is a marginal perturbation itself. Therefore the SU(4) Fermi liquid state is *robust* under the conditions discussed in the Introduction.
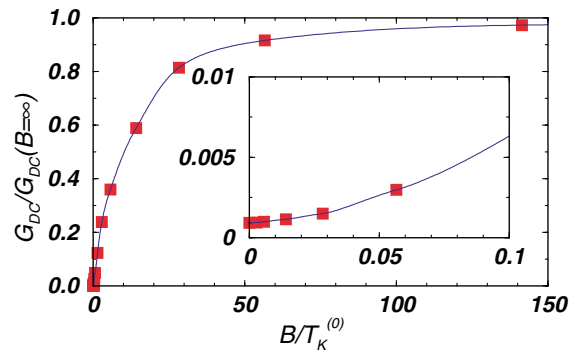

FIG. 3 (color online). The $T = 0$ conductance of DD system at $\omega = 0$ for $J = Q_z = 0.14$, $V_\perp = Q_\perp = 0.13$, $V_z = 0.14$ and different magnetic field values. Inset: small B limit of the conductance.
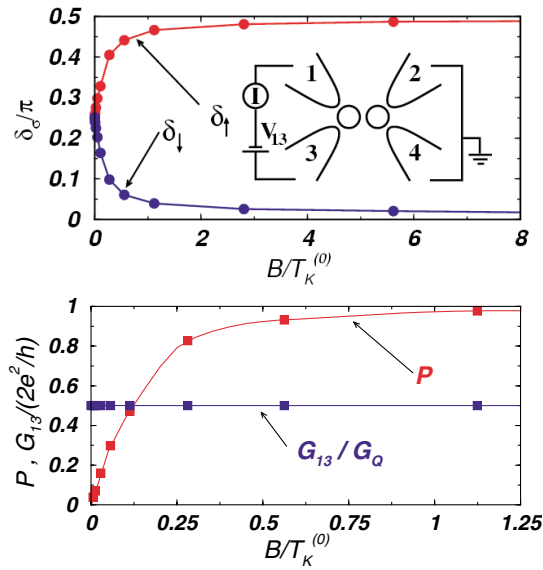
FIG. 4 (color online).   Top: Magnetic field dependence of the phase shifts for $t = \delta E \approx 0$. Bottom: Corresponding $T = 0$ dimensionless conductance and spin polarization of the current in the four-lead setup shown in the top inset.

Experimental accessibility.—For our scenario it is crucial to have large enough charging energy and level spacing $\tilde{E}_C, \Delta > T_K^{(0)} > t$. With today's technology it is possible to reach $\Delta \sim 2$–3 K. The dot-dot capacitance (and thus $\tilde{E}_C$ [7]) can be increased by changing the shape of the gate electrode separating the dots, using a columnar geometry as in Refs. [19,34], where the two-dimensional dots are placed on the top of each other, or placing an additional electrode on the top of the DD device [21]. We could not find a closed expression for $T_K^{(0)}$ in the general case. However, for a symmetrical DD $J \approx V_\perp \approx V_z \approx Q_\perp \approx Q_z \sim \Gamma/2\pi\tilde{E}_C$, provided that fluctuations to the $(0,0)$ state give the dominant contribution. Then we obtain $T_K^{(0)} \approx D e^{-1/4J}$ and $T_K(B = \infty) \approx$ cst$[T_K^{(0)}]^2/D$. Thus the value of J and thus $T_K^{(0)}$ can be tuned experimentally to a value similar to the single dot experiments. Indeed, an orbital Kondo effect has recently been observed [19].

Summary.—We have studied a DD system with large capacitive coupling close to its degeneracy point, in the Kondo regime. Using both scaling arguments and a nonperturbative NRG analysis, we showed that the simultaneous appearance of the Kondo effect in the spin and charge sectors results in an SU(4) Fermi liquid ground state with a phase shift $\pi/4$. Upon applying an external magnetic field, the system crosses over to a purely charge Kondo state with a lower T_K. In a four-terminal setup, the DD could thus be used as a spin filter with high transmittance. We further predict a large serial magnetoconductance at $T = 0$. The SU(4) behavior in this system is robust, and is experimentally accessible.

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